Waves

In addition to : (point) particles, extended objects (described by centre-of-mass motion + rotation), fields ($\vec{E}$, $\vec{B}$) we were confronted with electromagnetic waves, i.e., ($\vec{E}$, $\vec{B}$) fields that propagate in space-time.

Waves arise in simpler contexts: water waves, acoustic (sound) waves, waves on a string (standing vs travelling).

What exactly is a wave?

→ vibrating string (guitar, violin, piano)
some elastic mass is moving up/down

→ water wave : water droplets are sloshing up/down

→ acoustic wave : air is compressed/decompressed along the direction of travel

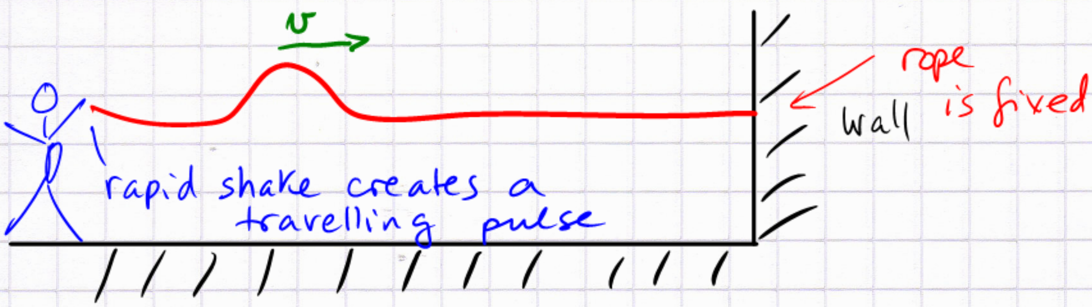
→ waves can travel, but don't have to (→ string ↓ fixed ends)

→ Some medium oscillates, oscillation can be transverse or longitudinal

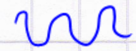
no medium → no sound propagation (demo!)

Electromagnetic waves are special: they propagate in vacuum! (induction makes this possible)

Start with travelling transverse wave pulse on a rope



Questions:

- what controls the propagation speed? (1)
- what happens to the pulse at the fixed end? (2)
- does the shape of the pulse change as it travels down the rope and back? (3)
- how do the particles move that make up the rope? (4)
- wave pulse  vs sinusoid  ? (5)

(1) → what can we change about the rope or string?

(A) thick vs thin string (piano strings?) ↳ or class. guitar

$$\mu = M/L = \frac{\text{mass}}{\text{length}}$$

(B) tensioning of string → tuning

tension force F_T is adjusted for fixed length string

Dimensional analysis: $\frac{F_T}{\mu}$ = accel. × length → (velocity)^2

wave propagation speed $v = \sqrt{\frac{F_T}{\mu}}$ Eq. (12.9)

transverse pulses move fast or slow according to this equation

Another example: shallow water (beach) → depth << wavelength λ

What can the propagation speed depend on?

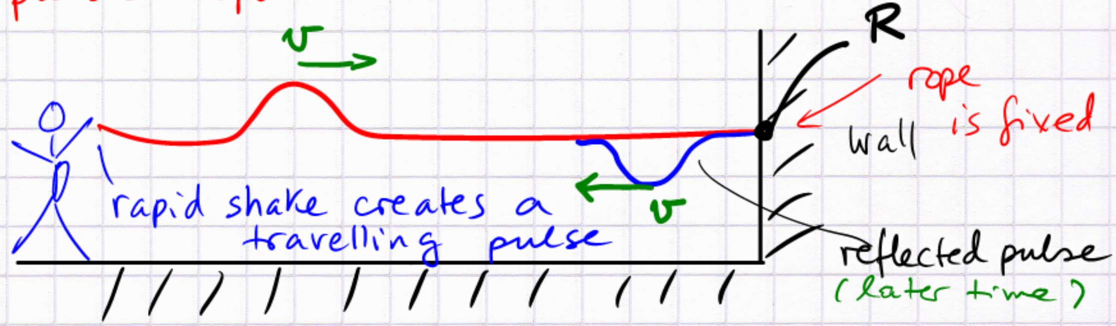
(A) depth of water, d (B) gravitational acceleration g

$v = \sqrt{gd}$ is the simplest formula on dimensional grounds

Q: does it work?

Q2 : How do pulses reflect ?

see Fig. 12.17 in G.
compare with 12.20



At point R the rope is fixed. Pulses reverse at R by flipping their shape vertically (Observation)

Q3 Apart from flipping the shape remains approximately the same.

However : water surface waves tend to dissipate (luckily for canoeists, when one stays away from a passing motor boat ...)

Yet, there exist water waves that propagate over huge distances (tsunami ; soliton waves)

→ bottom line : it's complicated, in general

Q4 : who moves, and in which direction ?

(1) small - amplitude waves on a string :

- mass segments Δm move up and down
- yet the pulse shape travels left ⟺ right

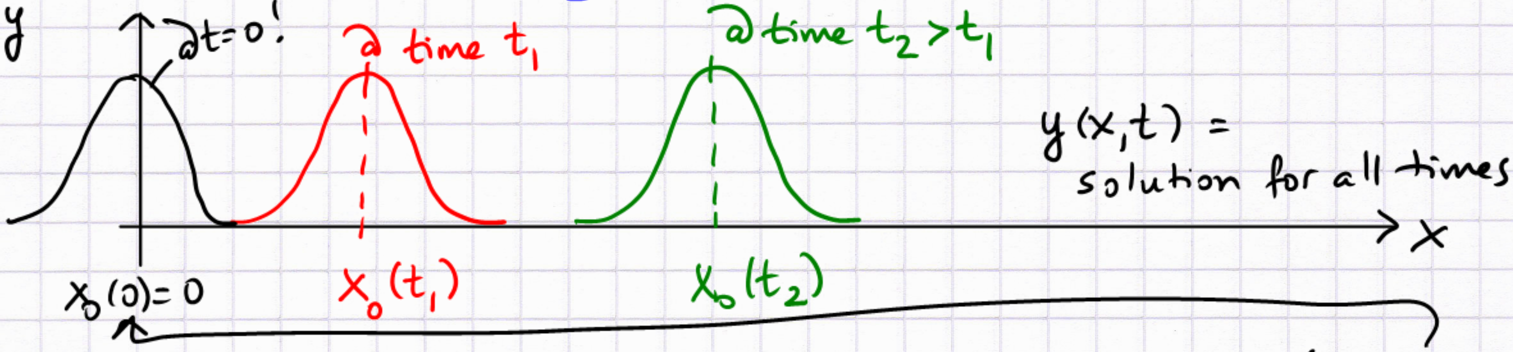
$\Big\}$!

(2) water surface waves : water droplets carry out a circular motion → up/down and left/right

- they do not propagate over larger distances

Travelling wave pulse

Q: how do we describe a "bump" travelling without the bump shape changing over time?



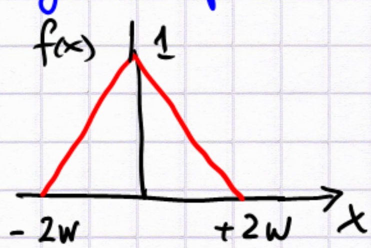
$x_0(t)$ = average position of bump $= v_w t$ $\quad$ $(=0 \text{ @ } t=0)$

$v_w = c$ = wave propagation speed

At $t=0$ simple : $y(x,0) = f(x)$ ← bump shape

Examples : $f(x) = \exp\left(-\left(\frac{x}{w}\right)^2\right)$ $\quad$ $w \sim$ width of bump

could also define triangle shape :



Question : how do we make $f(x)$ "move" in time ?

Answer : replace $f(x) \to f(x-ct)$

why ? The peak in f occurs for argument $= 0$,

argument $= x - ct = 0$ $\quad \therefore \quad x = ct$ yields x_0

Example :

$$f(x-ct) = \exp\left[-\frac{(x-ct)^2}{w_0^{\,2}}\right]$$

travels to the right (for $c > 0$)

Sound waves do not propagate in vacuum

they are longitudinal (along travel direction)

compressions + decompressions (rarefactions)

caused by the movement of

1) speaker membrane

2) instrument 'sound board' ↗ piano, harp, violin, guitar ↘ etc.

the strings are attached to a large-area board + make it vibrate

3) sound chords co-operate with oral cavity

4) wind instruments (pipe, flute, horn, etc.)

lips / mouthpiece create vibrations in air column of certain length

Longitudinal waves: particles oscillate in the direction of the wave propagation, but do not travel far

Common to all of this:

propagation of wave (energy transport)

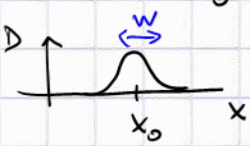
has little to do with the motion of the medium!

⇒ wave = new concept

it is more than the motion of the medium

Wave pulse vs wave train (Q5)

(1) Travelling wave pulse is charecterized by some shape function D; at fixed time $t_0 = 0$:

$D(x) =$  , e.g., $A e^{-\frac{(x-x_0)^2}{w^2}}$

Gaussian : $x \mapsto \exp(-x^2/w^2)$ $w \to$ width parameter

pulse is centered on location x_0

A = amplitude ($\exp(-0^2/w^2) = 1$)

For any time t : $A \exp\left(-\left[\frac{x-(x_0+vt)}{w}\right]^2\right)$

@ $t=0$ recover the above shape

@ $t>0$: centre location travels to $x_0 + vt$

The width parameter w is chosen as a constant

→ pulse preserves its shape

(2) An infinitely long wave train (Eq. 12.1 in G 12.2)

$D(x,t) = A \sin\left(2\pi f t - \frac{2\pi x}{\lambda}\right)$

is characterized by the same periodic (sine) function

in space (fix t, and observe) → period = wavelength λ

and in time (fix x, observe) → period = T, frequency $f = \frac{1}{T}$

We can simplify to $A \sin(\omega t - kx)$ $\omega \equiv 2\pi f$ circular frequency

Note: $\lambda \cdot f = c$ ← propagation speed ($= v$) $k \equiv \frac{2\pi}{\lambda}$ = wavenumber
(Eq. 12.2)

Later in math/phys we learn: a finite wave pulse can be decomposed into a mix (superposition) of sinusoidal wave trains (Fourier analysis). Arbitrary pulse = mix of many fixed frequency/wavelength periodic wave trains.