

# Standing waves $\rightarrow$ open end?

We derived standing-wave conditions for a string fixed at both ends:

- 1) left- and right-travelling sinusoidal waves of arbitrary frequency were added  $\rightarrow$  standing wave
- 2) node-conditions were imposed at  $x=0$  and  $x=L$ .

Quantization of modes:  $\lambda_n = \frac{2L}{n}$ ,  $f_n = \frac{c}{\lambda_n} \leftarrow v_w$   
 $n = 1, 2, 3, \dots$

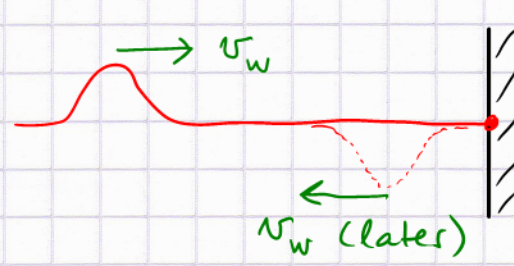
$f_1 =$  fundamental;  $f_2 = 2^{nd}$  overtone, etc.

Is this the only option? Can one end be open?

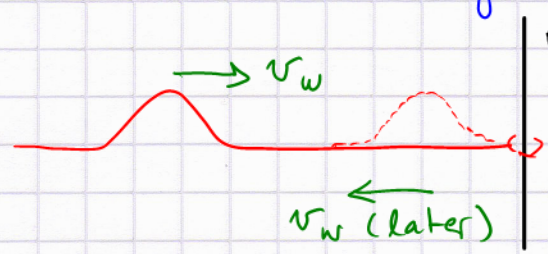
$\rightarrow$  Happens in wind instruments (end usually open, but can be "closed"  $\rightarrow$  change in frequency)

$\rightarrow$  Back to sending pulses on a rope:

Fig 12.17: end is fixed, pulse reflects (changes sign)



Alternative: end = ring on a slider, slides up/down when pulse arrives



RHS: not a node, but an antinode.

Add counterpropagating waves for this boundary: Chapter 13.3  $\rightarrow$  longitudinal!  $\rightarrow$  details for sound

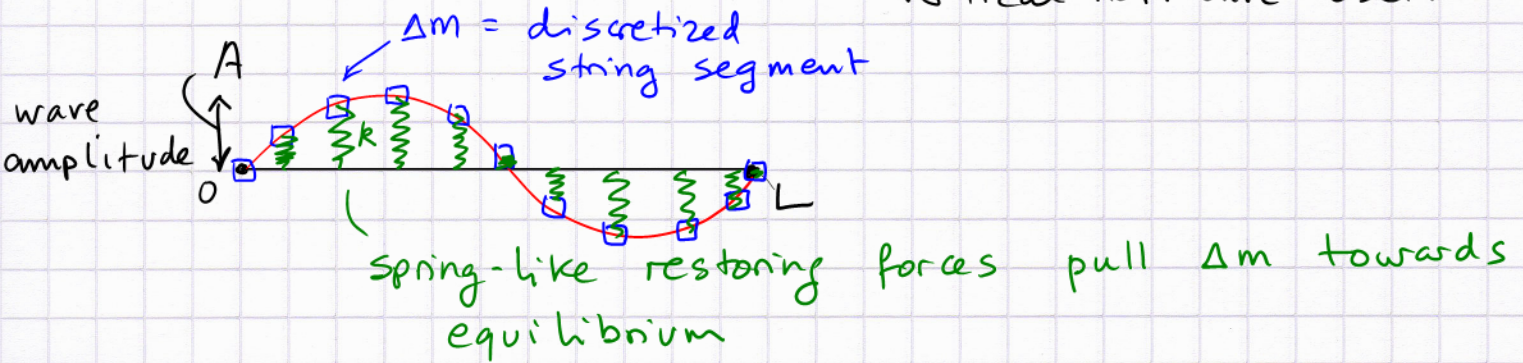


Wave intensity  $\sim$  amplitude squared

$I \sim A^2$  (3)

Plausibility argument:

transverse wave on a string  
mass segments  $\Delta m$  are  
vertical harmonic oscillators



Energy associated with transverse motion:

$$PE_{\max} \sim \frac{1}{2} k A^2$$

$$KE_{\max} = \frac{1}{2} \Delta m (v_y^{\max})^2$$

$$\text{Power} = \frac{\text{Energy}}{\text{Time}} \Rightarrow \text{Wave intensity} = \frac{P}{\text{area}} \sim A^2$$

Interesting:  $v_y^{\max}$  has nothing to do with  $v_w = c =$   
wave propagation speed!

$$D(x,t) = A \sin\left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda}\right) \quad v_w = \frac{\lambda}{T}$$

but

$$v_y(x,t) = \frac{\partial}{\partial t} D(x,t) = A \cos\left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda}\right) \left(\frac{2\pi}{T}\right)$$

$x$  is "constant"

$$v_y^{\max} = \frac{2\pi A}{T} = A\omega$$

wave intensity ( $\frac{\text{power}}{\text{area}}$ )  
is controlled by transverse  
(local) motion.

There is an independent (often much higher-velocity)  
energy scale associated with wave propagation.

EXM: wave guides photons moving with  $v=c$   
 $A$  controls how many photons/second; Photon energy:  $E = hf$  Planck