Planck distribution and photon model of radiation

The spectral curve derived by Planck and re-derived and interpreted by Einstein together with the photoelectric effect re-introduced a particle interpretation for radiation → photons

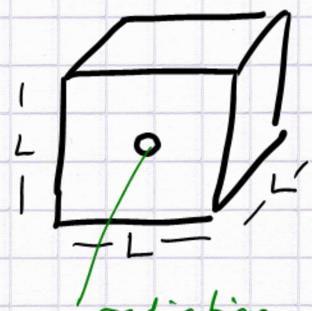
What was revolutionary about this?

Rayleigh had investigated black body radiation:

Box of dimension L^3
with a tiny hole.
Radiation enters.
Box walls are at some well defined temperature T, absorb and re-emit.

Hole = black body

walls are at temp. T



- trap radiation inside
- observe the hole once equilibrium is established

→ no reflection, absorption = emission (equilibrium)

What are the allowed radiation modes inside the box?

→ standing waves; products of 1d waves along xyz
(generalization of transverse waves on a string)

Want to observe: cavity walls are heated to temperature T ⇒ energy = kT

Q: How much radiation is coming out of the hole in a frequency window Δf at f: $[f, f+\Delta f)$

2-part question: 1) how many radiation modes exist in this frequency interval?

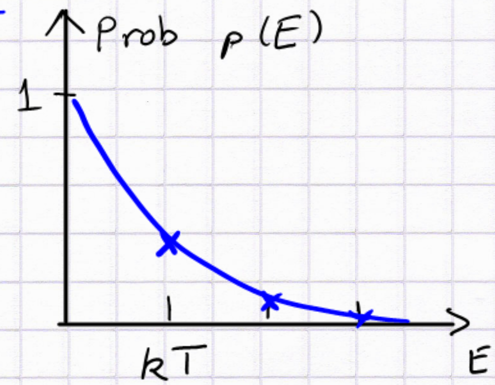
2) what is the average energy associated with these modes?

Q1: Rayleigh (and others) figured out:

mode density $\sim f^2$ $\left(f = \frac{c}{\lambda}\right)$, for larger λ the condition to "fit" nodes is harder to meet than for small λ

Q2: Take into account that at given thermal energy kT the probability to populate modes of higher energy E is small (Boltzmamn factor)

probability $p(E) = e^{-E/kT}$



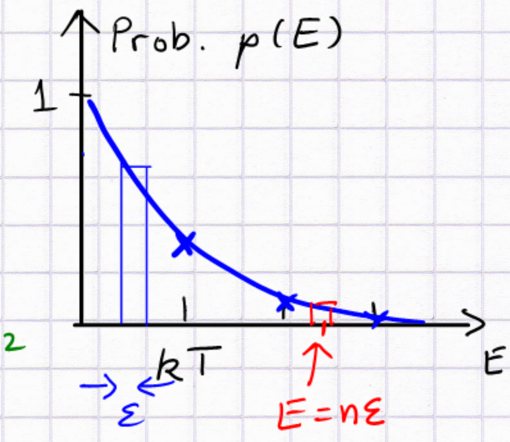
Classical Physics (Rayleigh-Jeans):

The energy of the radiation modes is a continuous variable, and is not related to frequency or wavelength → pure wave theory

Q: what is the average energy $\bar{E} = \langle E \rangle$ @ $[f, f+\Delta f)$?

→ statistical sum $\bar{E} = \dfrac{\int_0^\infty E\, e^{-E/kT}\, dE}{\int_0^\infty e^{-E/kT}\, dE}$ → estimate areas by rectangles of width ε; $E = n\varepsilon$

$$\bar{E} = \lim_{\varepsilon \to 0} \frac{\sum_{n=0}^{\infty} (n\varepsilon)\, e^{-n\varepsilon/kT} \cdot \varepsilon}{\sum_{n=0}^{\infty} e^{-n\varepsilon/kT} \cdot \varepsilon}$$



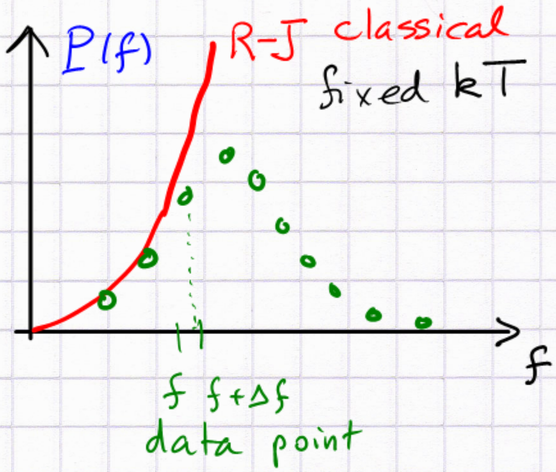
sums are calculated in closed form,

or use $\int_0^\infty e^{-x/y}\, dx = y$, $\int_0^\infty x e^{-x/y}\, dx = y^2$

to find $\bar{E} = \langle E \rangle = kT$

Independent of the radiation frequency (or wavelength) each mode has an average energy given by the wall temperature.

Measurements of hot ovens (metal smelter) disagreed with the result of combining:



mode density $n(f) \sim f^2$

mode energy $\mathcal{E}(f) = kT$

power density $P(f) \sim f^2 kT$

Apart from disagreement with experiment at moderate + high f:

This theory is sick, since the power density$/\Delta f$ when summed over all f should give the Stefan-Boltzmann total power result σT^4!

The problem shown for one value of kT exists at all T!

Einstein, following his work on the photoelectric effect began realizing that a pure wave theory couldn't work.

Radiation waves at frequency $f = \frac{c}{\lambda}$ have energy

$E = hf$ (and photon momentum $p = \frac{hf}{c} = \frac{h}{\lambda}$) associated with them

$\Rightarrow$ He explains Planck's finding:

Instead of allowing the modes to have continuous energies

$E_n = n\varepsilon$ with $\begin{matrix} n \to \infty \\ \varepsilon \to 0 \end{matrix}$ postulate $E_n = nhf$, $n = 0, 1, 2, \ldots$

This introduces a frequency ($f = \frac{c}{\lambda}$) dependence into the average energy associated with a mode interval Δf.

$$\bar{E} = \frac{\sum_{n=0}^{\infty} (n\varepsilon)\, e^{-n\varepsilon/kT}}{\sum_{n=0}^{\infty} e^{-n\varepsilon/kT}}$$

with $\varepsilon = hf$ $\longrightarrow$ $\dfrac{hf}{e^{hf/kT} - 1}$

and not $\varepsilon \to 0$ as $n \to \infty$!

sums + limits can be calculated in closed form!

What did this "quantum revolution" accomplish?

1) resolve a theoretical problem, called the ultraviolet catastrophe of classical radiation wave theory

2) bring back a particle interpretation of light (radiation), however, in a "duality" form.

3) provide evidence that the allowed states for particles making up the atomic walls of the cavity have quantized energies. → why?

The origin of the radiation in the cavity is radiation that got inside through the hole, then was absorbed and re-emitted by the walls (held at temperature T).

Planck carried his (more complicated) argument by demanding quantized radiators. The accelerating charges in the walls create radiation at energies $E_n = nhf$. This implies that they undergo transitions between discrete energy states: $E^{wall}_{high} - E^{wall}_{low} = E_n = nhf$

This is non-classical particle motion. A classical accelerated charge emits radiation continuously as it oscillates (frequency of radiation = oscillation freq.)

This finding motivates Bohr, Kramers, Heisenberg and others to work on the mechanics of atoms to explain discrete line spectra (atomic hydrogen → (~1860ies) Balmer formula $E_n = -\frac{E_0}{n^2}$, $n = 1, 2, \ldots$ $E_0 = 13.6\,\text{eV}$)

and in 1926 Heisenberg + Schrödinger independently formulate a <u>wave theory of matter</u>. dualism goes full circle