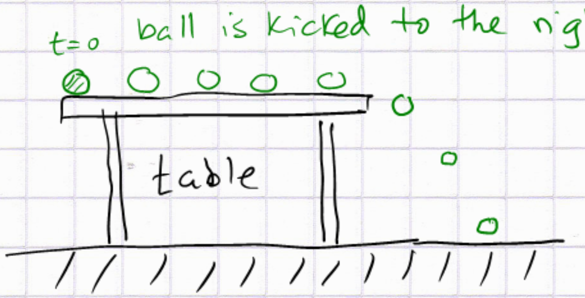
Description of motion = kinematics

Example

ball is kicked to the right, snapshots (video stream at some frame rate)



Q: how would we describe this motion?

8 snapshots at fixed time intervals t_i,

but really continuous motion. (truly continuous?)

↳ idealization to use functions $x(t), y(t)$

- At first ignore what made the ball move

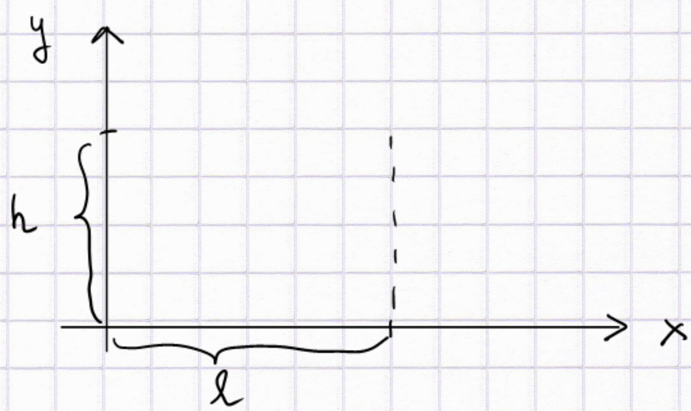
  (initial kick in the horizontal direction + vertical gravity)

- 2 dimensional motion: $x(t)$, $y(t)$; snapshots: $t = t_i$

- coordinate system

table: height h

length l

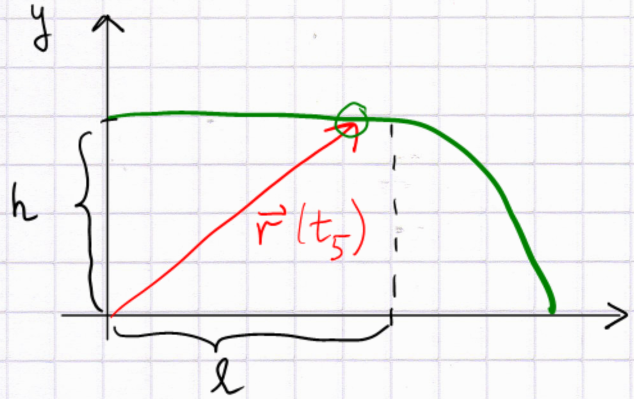


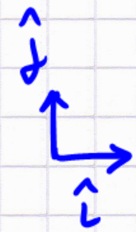
- first part of the motion:

  constant speed to the right (rolling ⇒ just model translation of the mass!)

  for $x(t) < l$

- then what? free fall in vertical direction, but translation to the right continues!





introduce position vector

$$\vec{r}(t) = x(t)\,\hat{\imath} + y(t)\,\hat{j}$$

series of snapshots is embedded in continuous $\vec{r}(t)$

Velocity vector: (in 2D)

$$\boxed{\vec{v}(t) \equiv \frac{d\vec{r}}{dt} = v_x(t)\,\hat{\imath} + v_y(t)\,\hat{j}}$$

$$v_x(t) \equiv \frac{dx}{dt} \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}, \qquad v_y(t) \equiv \frac{dy}{dt} \equiv \lim_{\Delta t \to 0} \frac{\Delta y}{\Delta t}$$

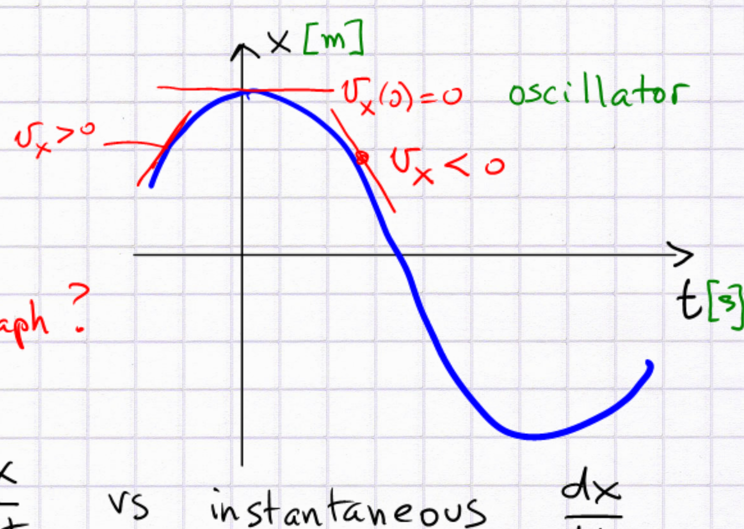
Speed: $v(t) = |\vec{v}(t)| = \sqrt{v_x(t)^2 + v_y(t)^2}$ (always ≥ 0)

1d motion: e.g., $v_y = 0$: $v_x(t)$ velocity $\neq$ speed due to sign.

$v_x > 0$ means travel along $\hat{\imath}$

$v_x < 0$ means travel against $\hat{\imath}$, along $-\hat{\imath}$.

Graphical representation:
(position vs time graph)



Q: how do we get $v_x(t)$ graph?
(slope of the curve)

- Distinguish: average velocity $\frac{\Delta x}{\Delta t}$ vs instantaneous $\frac{dx}{dt}$
  see Fig. 2.9