Exam prep

1) A particularly nice tone reaches your ear from a violin and has a wavelength of 39.1 cm. The room is on the warm side, and the speed of sound is 344 m/s. If the string's linear density is 0.600 g/m and the tension is 150 N, how long is the vibrating section of the violin's string?

Solution. A standing wave mode is produced by the string, then the soundboard produces a pressure wave in air. Distinguish the two propagation speeds!

$$v_{str} = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{150}{0.6\times10^{-3}}\ \frac{m}{kg}\ \frac{kg\ m}{s^2}}$$

$$= 500\ m/s$$

Note: not the same as v_{air}!

For the transverse wave on a string (assume fundamental)

$\lambda_s f = v_{str}$ The frequency is the same for the longitudinal air pressure wave and the string vibration.

Thus to obtain the frequency we look at the pressure wave:

$$\lambda_{sound} = \frac{v_{air}}{f} \quad \therefore \quad f = \frac{v_{air}}{\lambda_{sound}} \Rightarrow f = \frac{344}{0.391}\ \frac{\cancel{m}}{\cancel{m}}\ \frac{1}{s}$$

$$f = 879.8\ Hz$$

$\hookrightarrow$ 880 Hz.

Now determine the wavelength on the string:

$$\lambda_{str} = \frac{v_{str}}{f} = \frac{500}{880\ Hz}\frac{m}{s} = 0.568\ m$$

Now we need to consult the condition for string modes:

$\lambda_n = \frac{2L}{n}$. Let's assume that it is the fundamental ($n=1$)

$\therefore L = \frac{1}{2}\lambda_1$ $\quad\therefore L = 0.284\ m = 28.4\ cm$

2) How fast would a star have to travel to make $\lambda = 400$ nm violet light appear to be orange-red ($\lambda = 600$ nm)? Assume motion along the line of sight. Is the star moving away or towards earth?

A: This is a shift to longer wavelength $\rightarrow$ $\lambda f = c$ implies that the frequency is shorter $\rightarrow$ star moves away.

Go to a frame where earth is stationary (observer).

$$f_{obs} = f_{source}\frac{\sqrt{1 - v/c}}{\sqrt{1 + v/c}}$$

v = relative speed

$$\frac{c}{\lambda_{obs}} = \frac{c}{\lambda_{source}}\sqrt{\frac{1 - v/c}{1 + v/c}}$$

$$\frac{\lambda_{source}}{\lambda_{obs}} = \frac{400}{600} = \sqrt{\frac{1 - v/c}{1 + v/c}} \quad \therefore \left(\frac{2}{3}\right)^2 = \frac{1 - v/c}{1 + v/c}$$

③

$$\frac{4}{9}\left(1+\frac{v}{c}\right) = 1-\frac{v}{c} \quad \therefore \quad \frac{4}{9}-\frac{9}{9} = -\frac{v}{c} - \frac{4}{9}\frac{v}{c}$$

$$+\frac{5}{9} = \frac{v}{c}\left(\frac{13}{9}\right) \quad \therefore \quad \frac{v}{c} = \frac{5}{13} \qquad v = \frac{5}{13}c$$

$$v = 0.39\,c$$