Resonance

Consider a weakly damped oscillator

e.g.: mass on air track with some air resistance connected by a spring to a wall

or small-amplitude swing with linear air drag

Generically this is described by: $m x''(t) = -k x(t) - b x'(t)$

$$m x'' + b x' + k x = 0$$

If b is small (weak damping) this is an oscillator whose amplitude decreases slowly over many oscillations! (underdamped case, e.g., a mass attached to a slinky)

Natural frequency of oscillation: $\omega \sim \sqrt{\frac{k}{m}}$

$$= \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

Now ask: what happens if we push the mass with a sinusoidal force $F_x = F_0 \sin(\nu t)$

ν = circ. frequency of the drive

We have control over the strength of the drive F_0, and more interestingly: over the frequency $\nu_D = 2\pi f_D$ of the drive

Q: how does the spring-mass system (pendulum) respond to the drive?

A: initially, it will be complicated, but eventually the system responds with steady-state oscillations @ frequency ν_D amplitude A_0

Fig 11.28 shows how the amplitude of the steady-state oscillations changes with the drive frequency ν_D

For a very weakly damped mass-spring system:

- very strong response when $\nu_D \approx \omega \approx \underbrace{\sqrt{\frac{k}{m}}}_{\text{natural frequency}}$

  $\omega = \sqrt{\frac{g}{L}}$ for math. pendulum

- Some response (smaller amplitude) i.e., oscillations @ drive frequency when $\nu_D > \omega$ or $\nu_D < \omega$

$\therefore$ a weakly damped oscillator is very selective: strong response when the frequency of the drive matches

- a moderately damped oscillator: less peculiar, responds even when $\nu_D \neq \omega$, but doesn't respond well.

Electric oscillator example:

a radio set is exposed to signals from many stations (94.1 MHz, 96.3 MHz, 104.5 MHz, 107.? MHz, ...)

It has a tunable ω

It needs selectivity (low damping) to pick out only one carrier frequency, otherwise $\rightarrow$ jumbled audio from 2-3 stations

Mechanical oscillator example:

a wine glass (liquor glass) of good* quality:
*crystal glass, not plastic + glass.

- fill with some water (to ~1/3 height)
- wet your finger
- drive the finger around the rim while exploring the "rotation rate".

At some point the glass starts ringing

→ found the natural frequency of oscillation

→ increase intensity of the drive → glass may break.

Another application: pushing a child on a swing (or pumping yourself) → do it at the natural frequency → never slow down the system with outside force (push), always assist the motion

→ off resonance ($\nu_D \neq \omega$): this is not possible!