Inertia

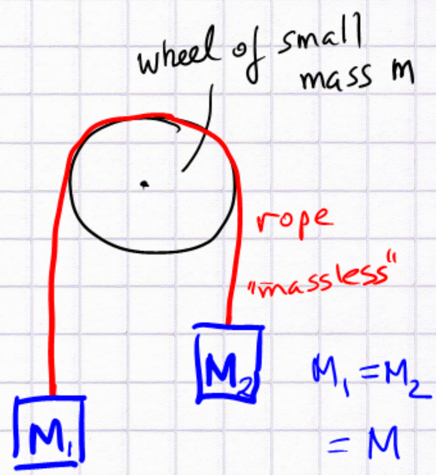
to understand it → need a force-free environment

e.g., perfect ice rink, air track [air hockey toy],
ball rolling on a track [sophisticated, friction makes ball roll!]
skier / snowboarder on groomed snow

i.e., air resistance + frictional forces ruled negligible
+ gravity compensated by a normal force

⇒ an object of mass m set in motion →
moves forever at constant speed

Example : Atwood machine



- gravity cancels
- wheel can be rotated without effort
  ($m << M_1 = M_2 = M$
- set a mass in motion (e.g. M_2 goes down)

→ $\dfrac{dy_1}{dt} = \text{const}$ $\qquad \dfrac{dy_2}{dt} = \text{const}$

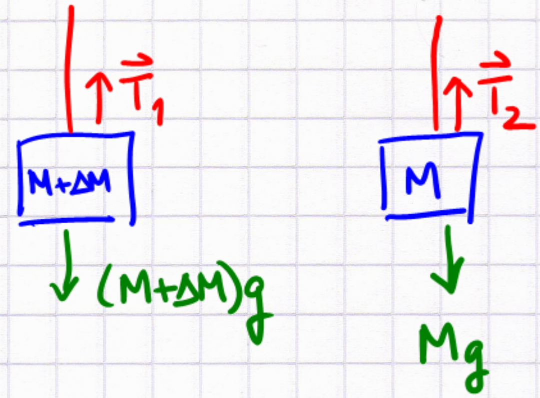
Motion is the result of inertia

$M_1 v_1 = \text{const}$ $\qquad a_1 = \dfrac{dv_1}{dt} = 0 \,!$

Detail: why is M_1 force-free? $M_1 g$ pulls down,
tension T_1 pulls up, but $T_1 = M_2 g$. Since $M_1 = M_2$ ⇒ no net force

Now add a small mass to M_1:

$$M_1 \to M_1 + \Delta M$$



We must have: $T_1 = T_2 = T$

(equal magnitude)

otherwise the rope would stretch!

Both masses experience an acceleration: (Newton's 2^{nd} law)

(1) $(M+\Delta M)\, a_1 = (M+\Delta M)\, g - T$

(2) $M a_2 = T - Mg$

$(T_1 = T_2 = T)$

M+ΔM moves down

M moves up

used vector info to define positive motion differently for M+ΔM vs M

Now solve by eliminating T:

(1): $T = (M+\Delta M)(g - a_1)$

(2) $T = M(g + a_2)$

also use $a_1 = a_2$, since rope doesn't stretch

$$(M+\Delta M)(g-a) = M(g+a) \quad \Big| -Mg \text{ on both sides}$$

$$\therefore \quad \Delta M g - (M+\Delta M)a = Ma$$

$$\therefore \quad (2M+\Delta M)\, a = \Delta M g \quad \therefore \quad \boxed{a = \frac{\Delta M}{2M+\Delta M}\, g}$$

reality check: 1) $\Delta M \to 0 \Rightarrow a = 0$ (as per page 1)

2) popular expt, since small ΔM implies easily observable small a motion

3) heavier masses $M \Rightarrow$ less a, since $\Delta M g$ has to provide the net force to make $2M + \Delta M$ move!