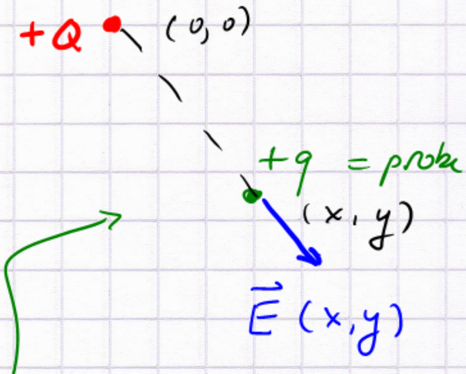
Electric field calculations

Single point charge $+Q$

place a probe charge $+q$ "virtually" at different locations (x, y), or $\vec{r} = x\hat{\imath} + y\hat{\jmath}$ $(+z\hat{k})$, compute the force there from Coulomb's law, and record $\vec{E}(\vec{r}) \equiv \frac{\vec{F}_c}{q}$

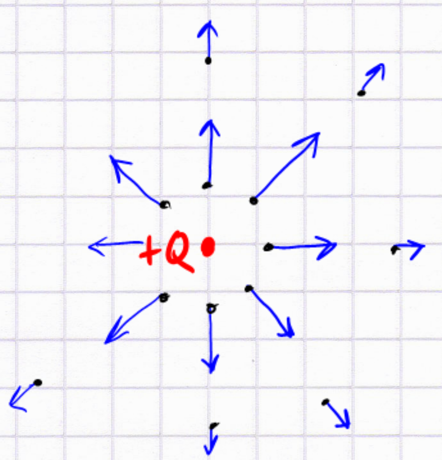


now repeat for different locations (x,y)

Main idea: Coulomb's law (like Newton's law of gravity) predicts an "action at a distance".

A given charge Q surrounds itself with an electric field $\vec{E}(\vec{r})$ ← means: $\vec{E}$ is defined for all locations $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

We can depict the $\vec{E}$ field by drawing vector arrows on a pre-selected grid of points (in the x-y plane)



$\vec{E}$ points radially outward

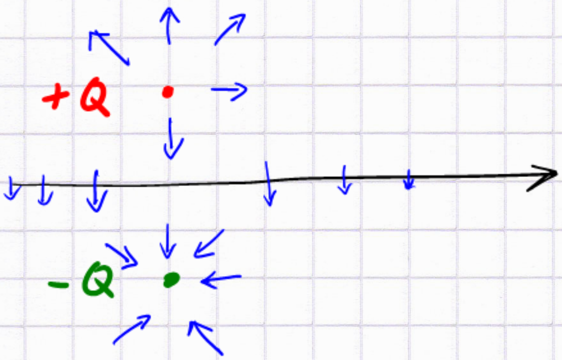
Length of arrows $\sim \frac{1}{r^2}$

$\vec{E}$ is defined everywhere, the arrows are just for a few points.

$\vec{E}$ - field direction: a positive charge q (probe) will accelerate in this direction, since $\vec{F} = q\vec{E}$

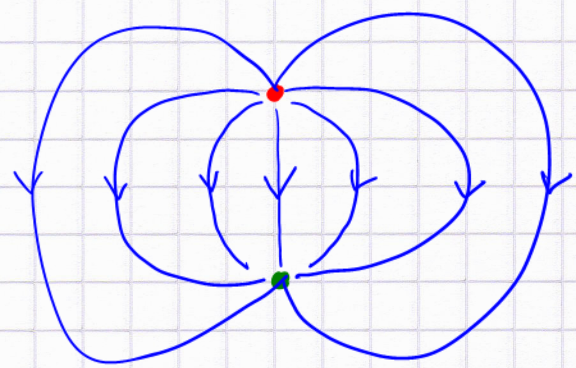
a negative charge $q_1 < 0$ accelerates in a direction OPPOSITE to $\vec{E}(x,y)$

Interesting field plot: a dipole



another view of $\vec{E}$:

through <u>field lines</u> (more continuous)



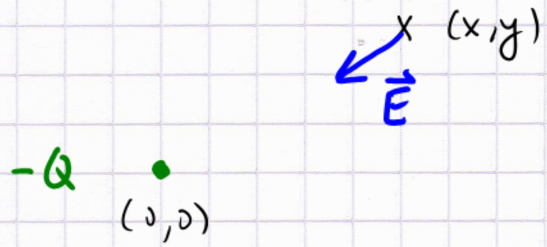
Field lines originate on + charge and terminate on − charge

Line density → field strength

Line direction: $\vec{E}$ is tangent to field lines

Use the software tool (RESOURCES page) charges-and-fields.swf to visualize $\vec{E}(x,y)$ for a collection of point charges!

Mathematical detail:



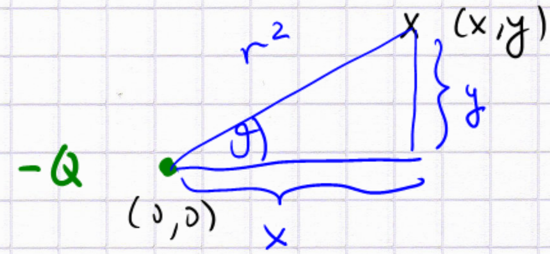
What is the field @ (x,y)?

$$r = \sqrt{(x-0)^2 + (y-0)^2} \qquad (z \equiv 0)$$

$$r^2 = x^2 + y^2$$

$$|\vec{E}| = \frac{KQ}{r^2}$$

$$(E_x, E_y) = ?$$



$$\cos\vartheta = \frac{x}{r} \qquad \sin\vartheta = \frac{y}{r}$$

$$E_x = |\vec{E}| \cdot (-\cos\vartheta)$$

↑ to the left, due to $-Q$ (!)

magnitude of charge! →

$$E_x = \frac{KQ}{r^2} \cdot \left(-\frac{x}{r}\right) = -\frac{KQx}{r^3}$$

$$E_y = |\vec{E}| \, (-\sin\vartheta) = \frac{KQ}{r^2}\left(-\frac{y}{r}\right) = -\frac{KQy}{r^2}$$

↑ downward

$$\therefore \quad \vec{E}(x,y) = -\frac{KQ}{r^3}\left(x\,\hat{\imath} + y\,\hat{\jmath}\right)$$

We used one (generic) point (x,y) and calculated the field in the entire plane!

NB: Field concept: **a charge Q in vacuum surrounds itself by an electric field**