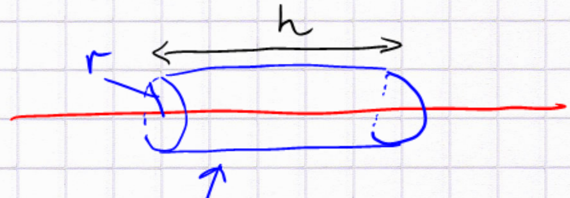
$\vec{E}$ fields : • line of charge
• charged plate → practical applications

Consider a long line of charge

linear charge density : $\frac{Q}{L}$



Gaussian surface = concentric cylinder of radius r

$\vec{E}$ points radially away from line. $\vec{E}$ is aligned with the normal vector for the surface

$\Rightarrow \vartheta = 0$; $\cos\vartheta = 1 \Rightarrow \Phi = E\,A$ (electric flux)

$$A = 2\pi r h \quad \Rightarrow \quad \Phi = EA = E(r)\, 2\pi r h = \frac{q}{\varepsilon_0}$$

How much charge q ? $\quad q = \frac{Q}{L} h$

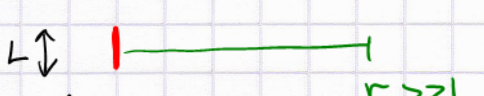
$$\therefore \quad E(r)\, 2\pi r h = \frac{1}{\varepsilon_0} \frac{Q}{L} h \quad \rightarrow \quad h \text{ cancels}$$

$$E(r) = \frac{1}{\varepsilon_0} \underbrace{\frac{Q}{L}}_{\eta} \frac{1}{2\pi r} = \frac{1}{2\pi\varepsilon_0} \frac{\eta}{r} = \frac{1}{2\pi\varepsilon_0} \frac{Q}{L r}$$

η = linear charge density

Here we assumed $L \gg r$, as we ignored flux through the circular end caps. By symmetry we assumed a radial $\vec{E}$, which is true when $L \to \infty$ (or $L \gg r$)

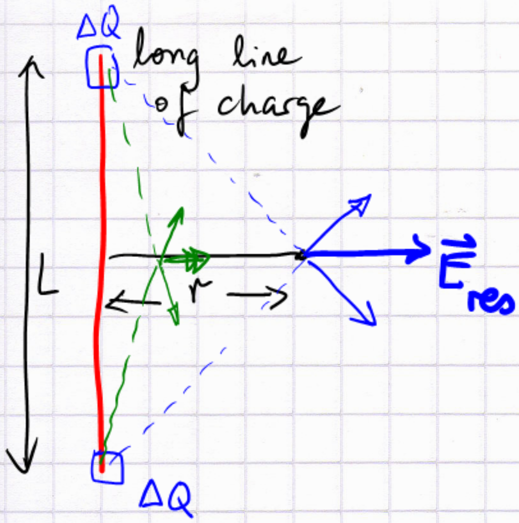
Some magic happened: for a short line of charge



total charge Q

we get $E(r) = \frac{|Q|}{4\pi\varepsilon_0 r^2}$

but when $L \gg r$: $E(r) = \frac{|Q|}{4\pi\varepsilon_0 L r}$



The far-away segments ΔQ contribute less → when $r \to 0$

arrows get shorter and angle gets more obtuse → smaller contribution to $\vec{E}_{res}$

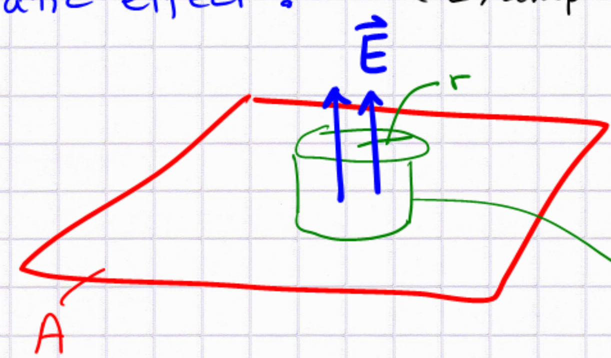
→ when we get closer to the line of charge:

the far-away charge segments make smaller contributions.

→ Close to the line of charge $E \sim \frac{1}{r}$, not $\frac{1}{r^2}$

→ a weaker fall-off!

For a charged plane of large size an even more dramatic effect: (Example 17.7)



charged plane

$\sigma = \frac{Q}{A} = \frac{\text{charge}}{\text{area}}$

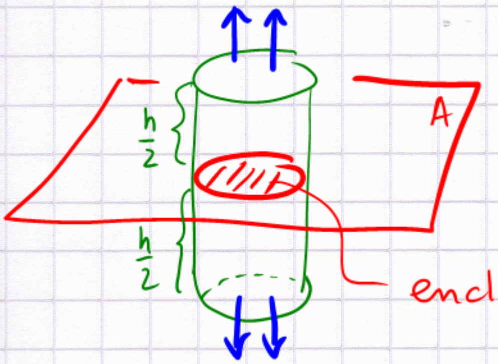
Gaussian surface = cylinder

Gaussian cylinder is small compared to A →

no flux through mantle, just through circular cap

charge enclosed: $q = \pi r^2 \frac{Q}{A} = \sigma \pi r^2$?

Not quite: to enclose charge we need a cylinder that intersects the plane



$$\Phi = (\pi r^2 + \pi r^2)\, E = 2\pi r^2 E$$

enclosed charge: $\frac{Q}{A}\pi r^2 = q$

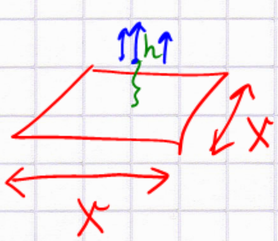
Gauss: $\Phi = 2\pi r^2 E = \frac{q}{\varepsilon_0} \Rightarrow 2\pi r^2 E = \frac{\frac{Q}{A}\pi r^2}{\varepsilon_0}$

$$\therefore \quad E_{plane} = \frac{1}{2\varepsilon_0}\frac{Q}{A} \equiv \frac{\sigma}{2\varepsilon_0}$$

As we change the height of the cylinder (get away from the plane), the $\vec{E}$ field does not change in strength (or orientation).

Such an $\vec{E}$ field is called uniform, or homogeneous

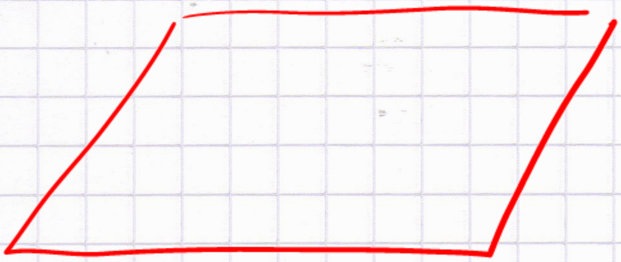
This is true as long as $h^2 \ll A$



finite charged plane (plate):

this property is satisfied at the centre for $h \ll x$

• $q < 0$ experiences a constant downward acceleration



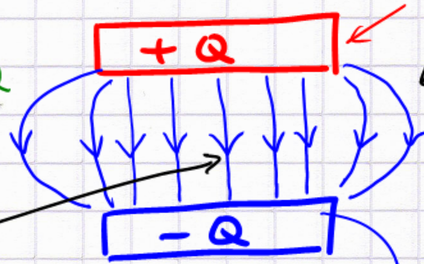
→ very much like gravity at the earth's surface

large plane, $\sigma = \frac{Q}{A}$

Parallel-plate capacitor

opposite, equal-magnitude charges $\pm Q$

$\sigma_+ = \frac{+Q}{A}$



fringe effect (weaker field)

uniform field

$\sigma_- = -\frac{Q}{A}$

$\perp$ to plates, strength independent of where one is (height or sideways displacement)

• field lines are equispaced

$$E = \frac{\sigma}{\varepsilon_0}$$ as both plates contribute $\frac{\sigma}{2\varepsilon_0}$ each

$$E = \frac{Q}{A\varepsilon_0}$$ A = area of one plate!

constant acceleration in vertical direction inside capacitor

→ parabolic trajectory (projectile motion)