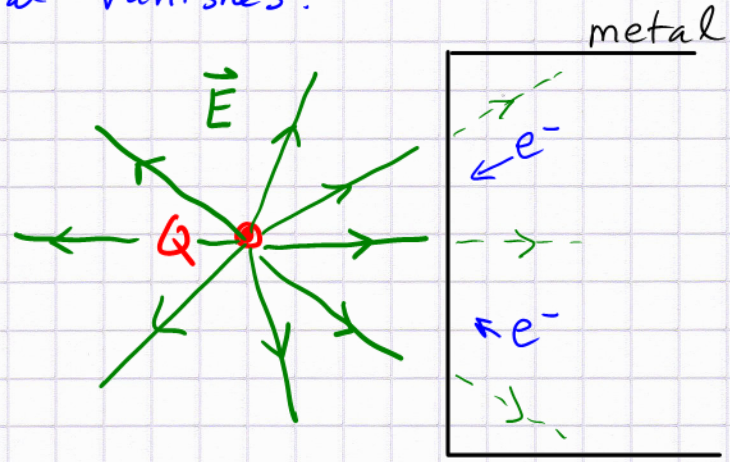
Electric Fields near Metals

- Metals form lattices of ions with one (sometimes two) e^- per atom becoming free to move. → conductors
- Crystals (salts) on the other hand tie up all atomic e^- to form bonds → insulators

Materials with tied up electrons will polarize when a charge gets close

Conductors respond more strongly: the freely movable charges inside the metal will move until the $\vec{E}$ field inside the metal vanishes.



$\vec{E}_Q$ due to Q causes electrons to move towards Q.

The $\vec{E}$ field due to the excess e^- cancels $\vec{E}_Q$ inside

Why does $\vec{E}_{el}$ have to cancel $\vec{E}_Q$?

As long as a net field exists inside the metal extra e^- will move to the surface

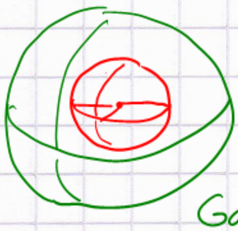
→ $\vec{E}_{net} = 0$ inside a metal (screen) → shielding (Faraday cage)

Net charge on a conductor? → has to be on the surface

Charged metal sphere → what is the $\vec{E}$ field ?

Using Gauss' law with probe surfaces = spheres surrounding metal sphere

sphere: charge Q, radius R



$\vec{E}$ on Gaussian sphere = radial

$\vec{E}$ parallel to normal on the surface

Gaussian probe sphere, radius r centered on the same origin

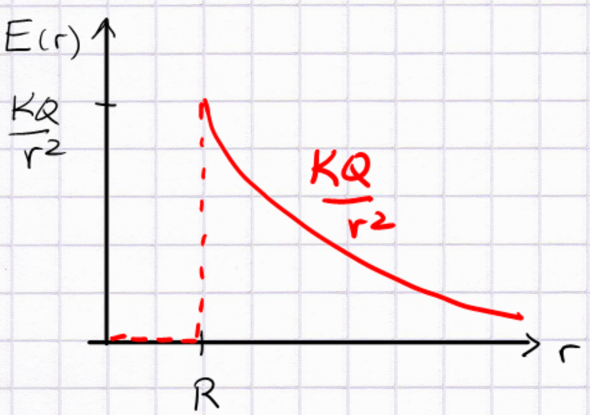
on Gaussian probe surface (arbitrary radius r, but $r \geq R$)

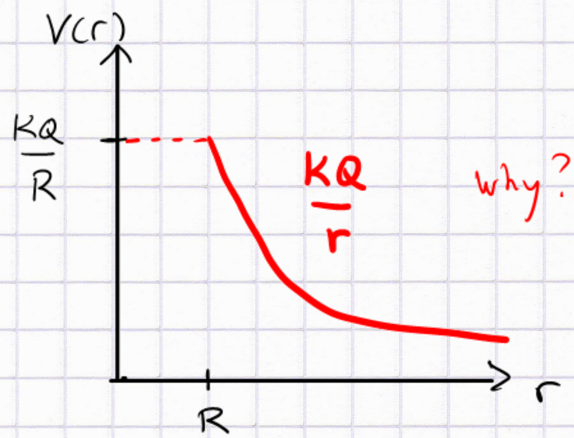
$$\Phi_E = E(r)\,A \quad , \quad A = 4\pi r^2$$

$$\Phi_E = \frac{Q}{\varepsilon_0} \quad \text{Gauss law}$$

$$E(r) = \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{KQ}{r^2}$$

the same, as if all Q was located at origin (point charge)





why? $E(r) = -\frac{dV}{dr}$

What is the $\vec{E}$ field inside the metal sphere ?

$E = 0$ inside, or the field would force electrons to move until $E = 0$ inside, i.e., until no more conduction electrons pushed around.

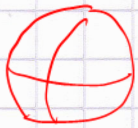
$\Rightarrow$ $E(r)$ for $r < R$ $= 0$

$V(r)$ for $r < R$ $=$ Const $= \frac{KQ}{R}$

NB: the discontinuous $E(r)$, and non-differentiable $V(r)$ at $r = R$ is the result of a simplification reality: surface has a width

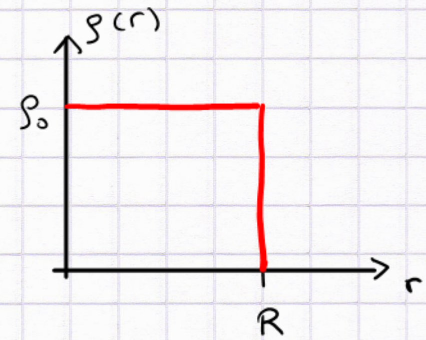
Q: what are $E(r)$ and $V(r)$ inside a nucleus?

high-Z (uranium: $Z = 92$) modeled by a sphere (reality: ellipsoid)



protons are densely packed

charge is distributed equally



$$\rho_0 = \frac{Ze}{V} = \frac{Q}{V}$$

$$V = 4\pi R^3$$

$$R \approx 1.2 A^{1/3} \text{ [fm]}$$

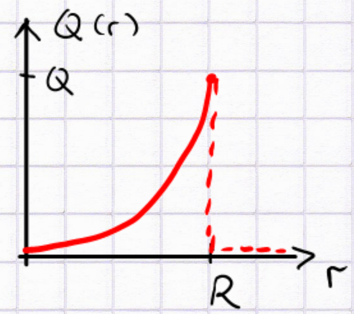
U^{238} is stable : $A = Z + N$ $\qquad R \approx 7.44 \times 10^{-15}$ m

U^{235} is radioactive

Now use Gaussian probe spheres with $r < R$ (inside nucleus)

Q: how much charge inside a sphere of radius r?

A: $Q(r) = \underbrace{\frac{Q}{V}}_{= \rho_0 = \text{charge density by volume}} \times \underbrace{4\pi r^3}_{\text{volume of Gaussian probe sphere}} = Q \cdot \left(\frac{r}{R}\right)^3$



Onion-layer model of a sphere:

layers at $r \lesssim R$ have much more volume than those for small r

⇒ hold more charge. Notice how misleading the graph for $\rho(r)$ is!!

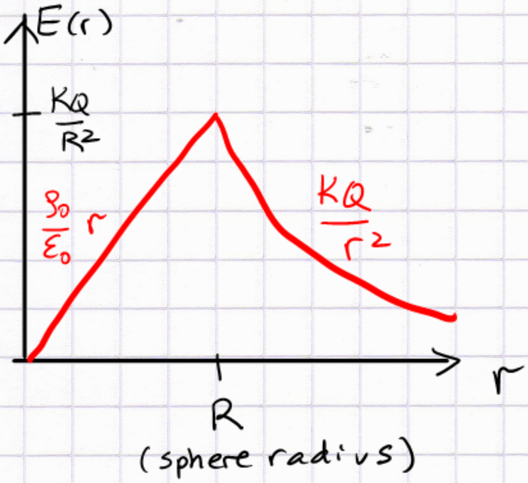
By Gauss' law: $\Phi_E = E(r)\, 4\pi r^2 = \frac{Q(r)}{\varepsilon_0} = \frac{Q r^3}{\varepsilon_0 R^3}$

$$E(r) = \frac{Q}{4\pi R^3} \frac{1}{\varepsilon_0} \frac{r^3}{r^2} = \frac{\rho_0}{\varepsilon_0} r$$ grows linearly

at the surface: $E(R) = \frac{Q}{4\pi R^3} \frac{1}{\varepsilon_0} R = \frac{KQ}{R^2}$ ← matches with "outside" result

Uniformly charged sphere (not a metal !!)

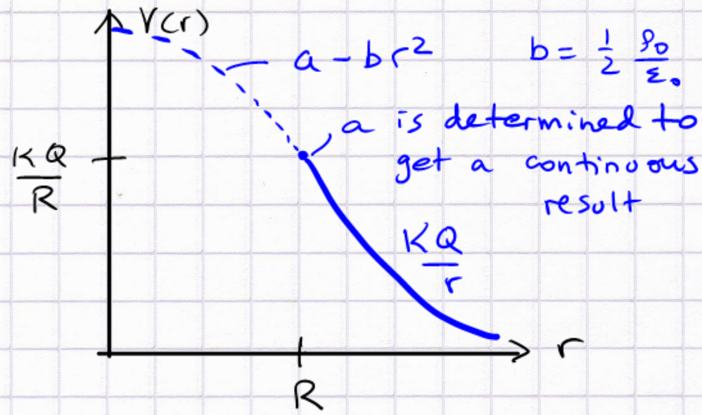
also: not a plastic sphere rubbed w. cloth



The potential inside this sphere is a quadratic

$V(r) \sim -r^2$ for $r < R$

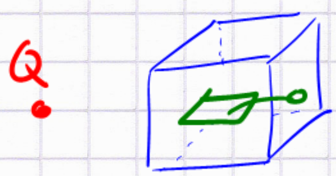
to satisfy $E(r) = -\frac{dV}{dr}$



Back to metals

Electronic devices (circuitry) do not respond well to external electric fields (particularly AC field from household wiring!)

a metal box



circuitry with connections via properly shielded (BNC) connectors (your stereo)

What happens when an outside (static) field is present?

Conduction electrons rush on the surface towards Q

→ adjacent surface = negatively charged
opposite surface = positively "

} inside the box $\vec{E} = 0$
to the right of the box: $\vec{E} \neq 0$