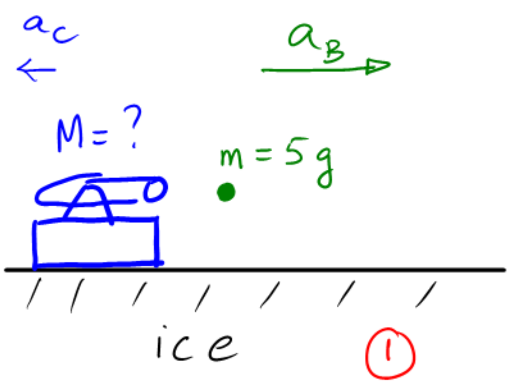
LAST NAME: STUDENT NR:

PHYS 1010 6.0: CLASS TEST 1

Time: 50 minutes; Calculators & formulae provided at the end = only aid; Total = 15 points.

1) A toy cannon is mounted on a hockey puck to shoot horizontally, and placed on perfect ice. When fired (by releasing a loaded spring) it accelerates a small ball of mass $m = 5$g inside its muzzle with $a = 30\text{m/s}^2$. The puck/cannon assembly recoils with an acceleration of $a = 0.25\text{m/s}^2$. What is the mass of the puck/cannon assembly?

a) [2] sketch the situation, and name and state the laws from which you can solve this problem.



3^{rd} law: action = reaction 0.5

Force on m to the right is equal and opposite to force on M (to the left)

2^{nd} law: $ma_B = F_B$ 0.5

$Ma_c = F_c$

b) [2] derive a formula for the mass of the puck/cannon assembly starting from the laws.

$$M \underbrace{= \frac{F_c}{a_c}}_{0.5} \underbrace{= \frac{F_B}{a_c}}_{1} \underbrace{= \frac{m a_B}{a_c}}_{0.5}$$

c) [1] calculate the mass of the puck/cannon assembly, and assess whether the answer is reasonable.

in SI: $M = \frac{5 \times 10^{-3} \cdot 30}{0.25} = 0.6$ 0.5 $M = 0.6$ kg 0.5

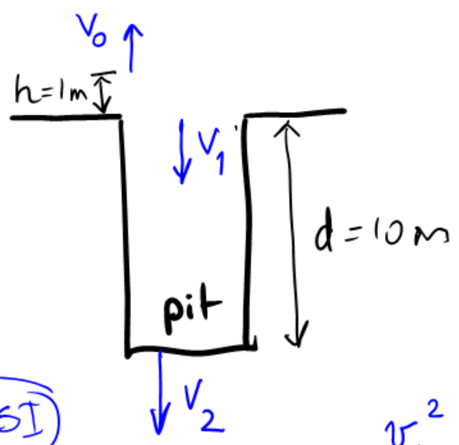
This seems reasonable, a puck weighs less than a pound, and we don't know much about the toy cannon

2) A ball is shot straight upward using a tennis racquet at height $h = 1\text{m}$ above ground level with speed v_0, while standing in front of a pit (big hole in the ground) of depth $d = 10\text{m}$. Air drag can be ignored, and use $g = 10\text{m/s}^2$.

a) [3] Sketch the situation indicating the velocity vectors at these points: release of ball (v_0), top of trajectory, passage of ground (v_1), hitting the bottom of the pit (v_2). Derive an expression for the speed of the ball as it returns and just misses the ground, (v_1), i.e., starts falling into the pit. Briefly, explain your steps.

x V = 0

1



in SI

This can be solved by constant-acceleration kinematics. Use

1 $v_f^2 = v_0^2 + 2a\,\Delta y$ where $a = -g$

if we measure upward-y as positive

This we see from the fact that the peak height y_{max} follows from $v_f = 0$

$\hookrightarrow \Delta y_{max} = -\dfrac{v_0^2}{(-2g)} = \dfrac{v_0^2}{2g}$ extra explanation

$v_1^2 = v_0^2 + 2(-g)(-1) = v_0^2 + 2g$ (in SI)

$= v_0^2 + 20\,\frac{m^2}{s^2}$ 1

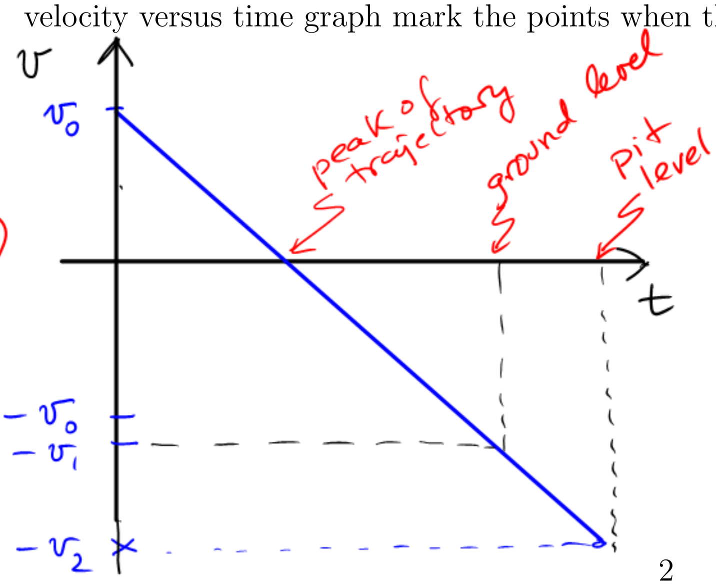
b) [1] What is the impact speed of the ball at the bottom of the pit, i.e., v_2.

$v_2^2 = v_0^2 + 2(-g)(-11) = v_0^2 + 22g$ (in SI) 1

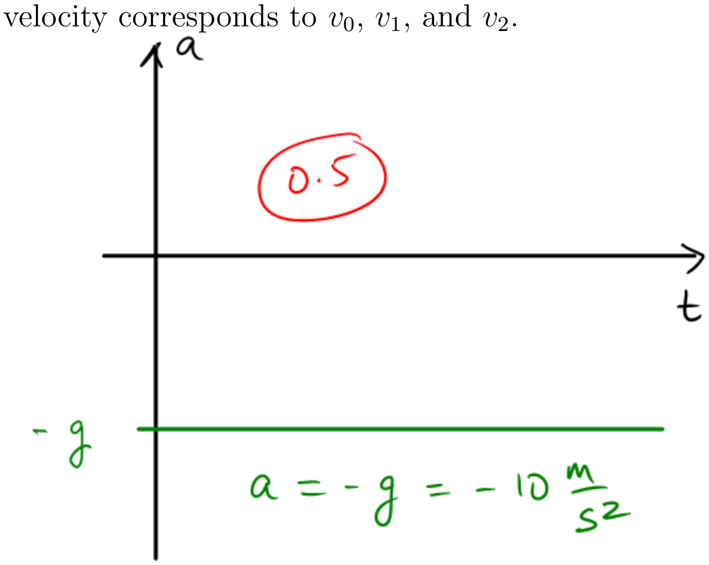
$= v_0^2 + 220\,\frac{m^2}{s^2}$

c) [1] Provide qualitative graphs of the velocity and acceleration of the ball as a function of time for the motion after the ball leaves the racquet, and just before it hits the ground. On the velocity versus time graph mark the points when the velocity corresponds to v_0, v_1, and v_2.

0.5

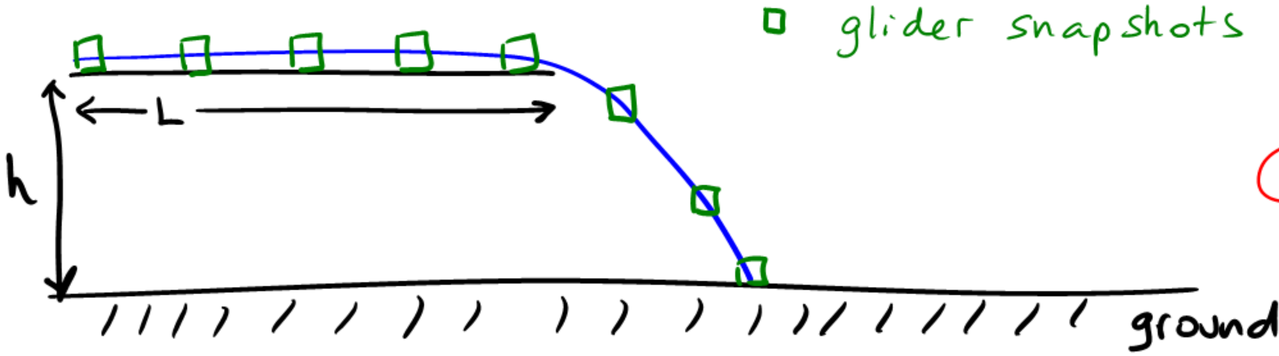


0.5



3) An air track of length L, is mounted at height h, and is operated with a glider set into constant-velocity motion (speed v_0) from left to right. Unfortunately, somebody removed the stopper at the right end.

a) [1] Provide a sketch of the situation, particularly of the trajectory of the glider up until it hits the ground.



1

b) [3] Use the kinematic equations to write the expressions describing the motion (position vector, velocity vector, acceleration vector).

$$\vec{a}(t) = a_x \hat{\imath} + a_y \hat{\jmath}.$$

$$\vec{v}(t) = v_x \hat{\imath} + v_y \hat{\jmath}$$

$$\vec{r}(t) = x \hat{\imath} + y \hat{\jmath}$$

= 1

on the track: $a_x = 0$, $a_y = 0$

off the track: $a_x = 0$, $a_y = -g$

on the track: $v_x = v_0$, $v_y = 0$

off the track: $v_x = v_0$, $v_y = -gt$

on the track: $x = v_0 t$, $y = h$

off the track: $x = v_0 t$, $y = h - \frac{1}{2} g t^2$

= 2

c) [1] Briefly explain, why in part (b) you have to distinguish between the two parts of the motion (glider is on the track vs glider leaves the track).

Newton's 2nd law: $m\vec{a} = \vec{F}_{net}$

on the track $\vec{F}_{net} = 0$, since a normal force suspends the glider ($M\vec{g} + \vec{N} = 0$) 0.5

off the track: $\vec{F}_{net} = M\vec{g}$ 0.5

FORMULA SHEET

$\vec{v}(t) = v_x(t)\ \hat{\mathbf{i}} + v_y(t)\ \hat{\mathbf{j}} = \frac{dx}{dt}\ \hat{\mathbf{i}} + \frac{dy}{dt}\ \hat{\mathbf{j}}$ $\quad \vec{a}(t) = a_x(t)\ \hat{\mathbf{i}} + a_y(t)\ \hat{\mathbf{j}} = \frac{dv_x}{dt}\ \hat{\mathbf{i}} + \frac{dv_y}{dt}\ \hat{\mathbf{j}}$

$v_\mathrm{f} = v_\mathrm{i} + a\Delta t \quad s_\mathrm{f} = s_\mathrm{i} + v_\mathrm{i}\Delta t + \frac{1}{2}a\Delta t^2 \quad v_\mathrm{f}^2 = v_\mathrm{i}^2 + 2a\Delta s \qquad g = 9.8\ \mathrm{m/s^2}$

$v(t_\mathrm{f}) = v(t_\mathrm{i}) + \int_{t_\mathrm{i}}^{t_\mathrm{f}} a(t)\ dt \quad s(t_\mathrm{f}) = s(t_\mathrm{i}) + \int_{t_\mathrm{i}}^{t_\mathrm{f}} v(t)\ dt$

$f(t) = t \quad \frac{df}{dt} = 1 \quad F(t) = \int f(t)\ dt = \frac{t^2}{2} + C$

$f(t) = a \quad \frac{df}{dt} = 0 \quad F(t) = \int f(t)\ dt = at + C \quad F(t) =$ anti-derivative = indefinite integral

area under the curve $f(t)$ between limits t_1 and t_2: $F(t_2) - F(t_1)$

$x^2 + px + q = 0$ factored by: $x_{1,2} = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}$