LAST NAME: STUDENT NR:

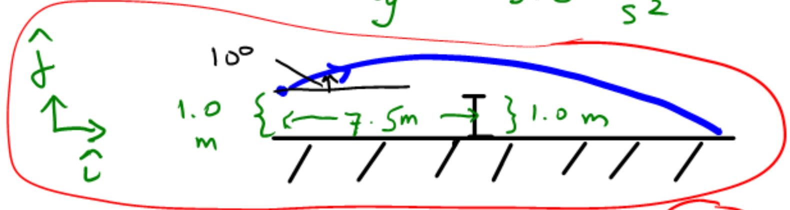
PHYS 1010 6.0: CLASS TEST 2

Time: 50 minutes; Calculators & formulae provided at the end = only aid; Total = 20 points.

1) A tennis ball is hit at a height of 1.0 m above the ground without spin. The ball leaves the racquet with a speed of 15 m/s with an angle of 10 degrees above the horizontal. The net is ~~[illegible]~~ metres away from the player, and has a height of 1.0 m. Does the ball clear the net? If so, by how much? If not, by how much does it miss? Start your solution with a pictorial representation.

7.5

$a_y = -9.8\ \frac{m}{s^2}$



1.0

x - motion : $a_x = 0$

$v_x(t) = v_{0,x}$

$x(t) = v_{0x}\, t$ 0.5

y - motion : $a_y = -9.8\ \frac{m}{s^2}$; $v_y(t) = v_{0y} - g t$

$y(t) = y_0 + v_{0y}\, t - \frac{1}{2} g t^2$ 0.5

$v_0 = 15\ m/s$, $\vartheta = 10°$ $\therefore$ $v_{0x} = v_0 \cos\vartheta = 15 \cdot 0.985\ \frac{m}{s} = 14.8\ \frac{m}{s}$
0.5 $= 15.0\ \frac{m}{s}$

$v_{0y} = v_0 \sin\vartheta = 15 \cdot 0.174\ \frac{m}{s} = 2.60\ \frac{m}{s}$
0.5 $= 2.6\ m/s$

x-motion yields time of arrival at the net distance :

$x_f = 7.5\,m = v_{0x} t_f$ $\therefore$ $t_f = \frac{x_f}{v_{0x}} = \frac{7.5}{15.0}\, s = 0.5\ s$

0.5

The vertical motion yields the height :

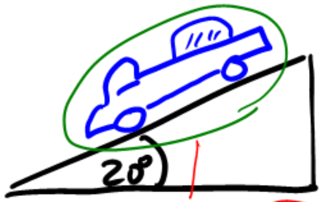
$y(t_f) = 1.0\,m + 2.6\ \frac{m}{s} \times 0.50\,s - \frac{1}{2}\, 9.8\ \frac{m}{s^2} \times 0.50^2\ s^2 = 1.075\,m$
$= 1.1\,m$

1.0

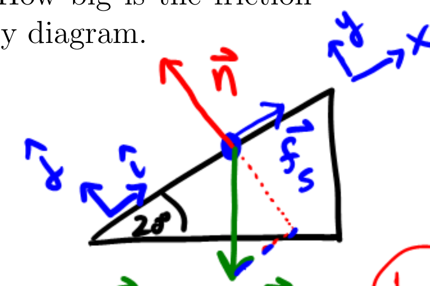
The tennis ball goes over the net by 10 cm
0.5

(it does not get caught, since its radius is < 10 cm)

2) A 15,000 kg truck with good brakes is parked on a 20° slope. How big is the friction force on the truck? Start with a pictorial representation and a free-body diagram.



gravity (0.5)
normal force
static friction



$\vec{W} = m\vec{g}$ (1.0)

(+0.5) for accuracy (vectors adding to ≈ 0)

Choosing x along the direction of the road we find that

$\vec{F}_{net} = \vec{0}$ ∴ $F_{net,x} = 0$

$F_{net,y} = 0$ 1.0

∴ $F_{net,x}$: $mg_{\parallel} = f_s$ $F_{net,y}$: $mg_{\perp} = n$

Given that the truck is stationary (it's not slipping):

$$f_s = \underbrace{mg_{\parallel} = mg\sin\vartheta}_{1.0} = 15 \times 10^3\ \text{kg} \times 9.8\ \frac{\text{m}}{\text{s}^2} \times 0.34$$

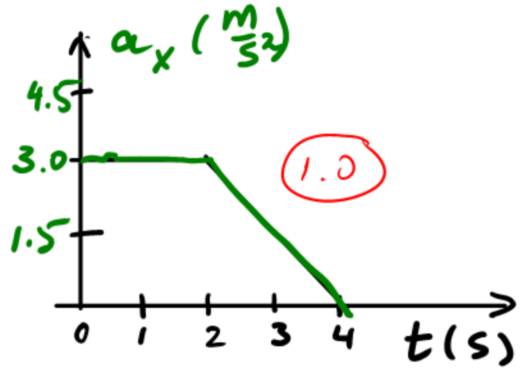
$$= 50 \times 10^3\ \text{N} = 5 \times 10^4\ \text{N}$$ (1.0)

3) A 2.0 kg object initially at rest at the origin is subjected to the time-varying force shown in the figure. What is the object's velocity at $t = 4$s? Start with an acceleration versus time graph.

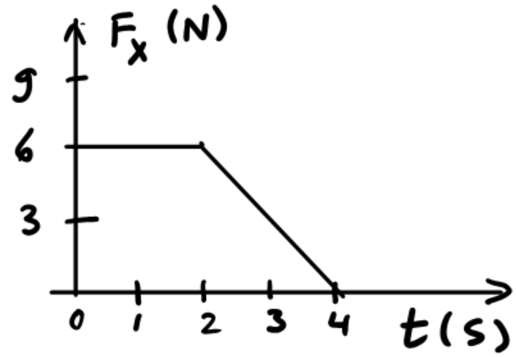
Using Newton's 2nd law:

$a_x = \frac{F_x}{m}$ (1.0)

we graph



(1.0)



The velocity $v_x \equiv v$ is found from area under the curve: ($t_f = 4$s)

$$v_f = v_i + \int_0^{t_f} a(t)dt \quad ∴ \quad v_f = \left(0 + 2 \times 3 + \frac{1}{2} \times 2 \times 3\right)\frac{m}{s}$$

(1.0)

$$= 9\ m/s$$ (1.0)

From geometry:

area = rectangle + triangle (1.0) or other explanation, e.g., using Calculus

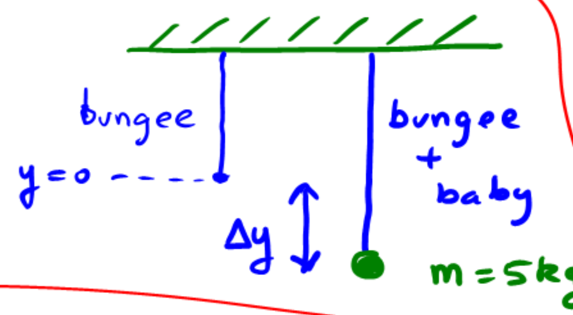
4) A bungee spring is hanging from the ceiling. Attaching a 5.0 kg baby to the spring causes it to stretch 40 cm in order to reach equilibrium. Ignore damping due to air drag or internal friction in the bungee spring for parts (a-c).

a) what is the spring constant? (start with a pictorial representation and a free-body diagram)

0.5

$\vec{F}_H = k\Delta y\, \hat{j}$

$\vec{w} = m\vec{g} = -mg\,\hat{j}$



equate magnitudes:

$$mg = k\,\Delta y$$

$$\underbrace{k = \frac{mg}{\Delta y} = \frac{5.0 \times 9.8}{0.4}\,\frac{N}{m} = 122.5\,\frac{N}{m} = 120\,\frac{N}{m}}_{0.5}$$

b) From equilibrium the baby is pulled down 10 cm (feet are barely reaching the floor) and released. Derive the period of oscillation from Newton's 2^{nd} law, given the info on the formula sheet and calculate it.

Choose y coordinate direction: $\uparrow \hat{j}$

$$ma_y = -mg - k(y - y_0) = -mg - ky$$

(when $y_0 = 0$ is the original equilibrium)

$$a_y = -g - \frac{k}{m}y$$

1.0

$$\frac{d^2y}{dt^2} = -g - \frac{k}{m}y(t)$$

is solved by $y(t) = y_1 + A\cos(\omega t)$

where $\omega = \sqrt{\frac{k}{m}}$ and $y_1 = -\frac{m}{k}g$

why? $y''(t) = -A\omega^2 \cos\omega t = \text{LHS}$

$$\text{RHS} = -g - \frac{k}{m}\left(-\frac{m}{k}g + A\cos\omega t\right) = A\frac{k}{m}\cos\omega t \quad \therefore\ \omega^2 = \frac{k}{m}$$

0.5

Thus, $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{120\ N/m}{5.0\ kg}} = \sqrt{24}\ \frac{1}{s} = 4.9\ \frac{rad}{s}$

$f = \frac{\omega}{2\pi} = 0.78\,\frac{1}{s}$ $\left(\frac{\text{oscillations}}{\text{second}}\right)$

0.5

This is not unpleasant ⟹ "Jolly Jumper"

c) Derive the maximum speed formula for harmonic oscillations, and calculate its value for the baby.

Use $\Delta y(t) = A(\cos \omega t + \phi)$ ($\phi = 0$ is ok)

$v_y(t) = \frac{d}{dt} \Delta y(t) = -A \sin(\omega t + \phi) \cdot \omega$ 0.5

max value when $\sin(\omega t_m + \phi) = \pm 1$

$\therefore \quad v_y^{max} = A\omega = A\sqrt{\frac{k}{m}}$ 0.5

The amplitude is given by the initial displacement from the "new" equilibrium with gravity, $A = 0.1$ m.

$v_y^{max} = 0.1 \times 4.9 \frac{m}{s} = 0.49 \frac{m}{s}$ 0.5

Not unpleasant for the "jolly jumper"

d) Formulate Newton's law for the problem while including damping due to a linear drag force.

$y_0 = 0$ also ok

v_y is also ok

$m y''(t) = -mg - k(y - y_0) - d y'$ 0.5

FORMULA SHEET

$v(t_\mathrm{f}) = v(t_\mathrm{i}) + \int_{t_\mathrm{i}}^{t_\mathrm{f}} a(t)\ dt \qquad s(t_\mathrm{f}) = s(t_\mathrm{i}) + \int_{t_\mathrm{i}}^{t_\mathrm{f}} v(t)\ dt$

$v_\mathrm{f} = v_\mathrm{i} + a\Delta t \qquad s_\mathrm{f} = s_\mathrm{i} + v_\mathrm{i}\Delta t + \frac{1}{2}a\Delta t^2 \qquad v_\mathrm{f}^2 = v_\mathrm{i}^2 + 2a\Delta s \qquad g = 9.8\ \mathrm{m/s^2}$

$f(t) = t \quad \frac{df}{dt} = 1 \quad F(t) = \int f(t)\ dt = \frac{t^2}{2} + C$

$f(t) = a \quad \frac{df}{dt} = 0 \quad F(t) = \int f(t)\ dt = at + C \qquad F(t) =$ anti-derivative = indefinite integral

area under the curve $f(t)$ between limits t_1 and t_2: $F(t_2) - F(t_1)$

$x^2 + px + q = 0$ factored by: $x_{1,2} = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}$

uniform circular m. $\vec{r}(t) = R(\cos\omega t\ \hat{\mathbf{i}} + \sin\omega t\ \hat{\mathbf{j}})$; $\vec{v}(t) = \frac{d\vec{r}}{dt} = ...$; $\vec{a}(t) = \frac{d\vec{v}}{dt} = ...$

$\exp' = \exp$; $\sin' = \cos$; $\cos' = -\sin$. $\quad \frac{d}{dx}[f(g(x))] = \frac{df}{dg}\frac{dg}{dx}$; $\quad (fg)' = f'g + fg'$

$m\vec{a} = \vec{F}_\mathrm{net}$; $\quad F_G = \frac{Gm_1m_2}{r^2}$; $g = \frac{GM_E}{R_E^2}$; $R_E = 6370$ km; $G = 6.67 \times 10^{-11}\frac{\mathrm{Nm^2}}{\mathrm{kg^2}}$; $M_E = 6.0 \times 10^{24}$kg

$f_\mathrm{s} \le \mu_\mathrm{s} n$; $\quad f_\mathrm{k} = \mu_\mathrm{k} n$; $\quad f_\mathrm{r} = \mu_\mathrm{r} n$; $\quad \mu_\mathrm{r} << \mu_\mathrm{k} < \mu_\mathrm{s}$. $\qquad F_H = -k\Delta x = -k(x - x_0)$.

$\vec{F}_\mathrm{d} \sim -\vec{v}$; linear: $F_\mathrm{d} = dv$; quadratic: $F_\mathrm{d} \approx 0.25Av^2$; $\quad A =$ cross sectional area

$x(t) = A\cos(\omega t + \phi)$; $\quad \omega = \frac{2\pi}{T} = 2\pi f$; $\quad v_x(t) = ...$; $\quad v_\mathrm{max} = ...$