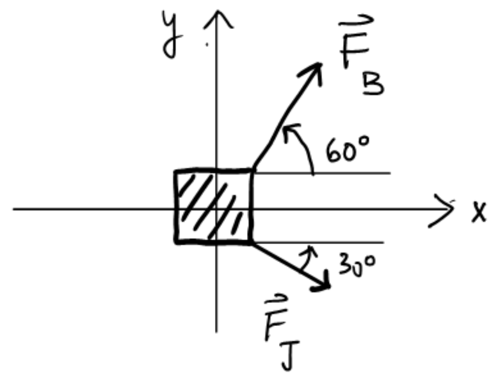
LAST NAME: STUDENT NR:

PHYS 1010 6.0: CLASS TEST 2

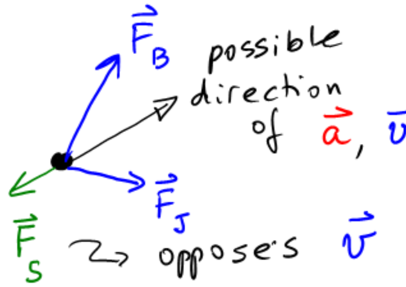
Time: 50 minutes; Calculators & formulae provided at the end = only aid; Total = 20 points.

1)[5] A crate ($m = 1.0 \times 10^2$ kg) needs to be pulled across a smooth floor by John and Bob as shown in the figure. The friction coefficients are known as $\mu_s = 0.25, \mu_k = 0.10$. The crate location at time $t = 0$ is shown, John pulls with $F_J = 1.0 \times 10^2$ N, and Bob with $F_B = 2.0 \times 10^2$ N at the angles indicated. Provide answers for $x(t)$ and $y(t)$, i.e., for the position vector of the motion. Start with a free-body diagram (include friction!). Will the crate move?



Free-body diagram

m



$\vec{F}_S \rightsquigarrow$ opposes $\vec{v}$

1

(all parts reasonable, not quantitative)

$\vec{F}_B + \vec{F}_J$ give direction of motion

$\vec{F}_s, \vec{F}_k$ opposes " "

Determine the maximal static friction force:

$$F_{s,max} = \mu_s N = \mu_s m g = 0.25 \times 100\,\text{kg} \times 9.8\,\frac{\text{m}}{\text{s}^2} = 245\,\text{N}$$ 0.5

The net pulling force: $\vec{F}_p = \vec{F}_J + \vec{F}_B$ 0.5

Does it exceed $F_{s,max}$? We need: $|\vec{F}_p| = \sqrt{F_{p,x}^2 + F_{p,y}^2}$ 0.5

$$F_{pull,x} = 200 \cos(60°) + 100 \cos(30°) \quad \text{in SI (N)}$$
$$= 100 + 86.6 = 186.6\,\text{N}$$ 0.5

$$F_{pull,y} = 200 \sin(60°) - 100 \sin(30°) = 123.2\,\text{N}$$ 0.5 is positive

$$\boxed{\vartheta_{motion} = \tan^{-1}\left(\frac{F_{pull,y}}{F_{pull,x}}\right) = \tan^{-1}(0.660) = 33.4° \text{ (wrt +ve x-axis)}}$$ optional

$$F_p = |\vec{F}_p| = \sqrt{186.6^2 + 123.2^2}\,\text{N} = 224\,\text{N}$$ 0.5

1 $\therefore$ No motion, since static friction is not overcome $x(t) = 0$, $y(t) = 0$

2) [5] Derive the formula for the centripetal acceleration ($a_{\text{cp}} = \frac{v^2}{r}$) from the position vector describing uniform circular motion (formula sheet!), and show the direction for the acceleration vector.

$$\vec{r}(t) = R\cos\omega t\ \hat{\imath} + R\sin\omega t\ \hat{\jmath}$$

$$\vec{v}(t) = \frac{d\vec{r}}{dt} = -R\omega\sin\omega t\ \hat{\imath} + R\omega\cos\omega t\ \hat{\jmath} \quad 0.5$$

$$\vec{a}(t) = \frac{d\vec{v}}{dt} = -R\omega^2\cos\omega t\ \hat{\imath} - R\omega^2\sin\omega t\ \hat{\jmath} \quad 0.5$$

$$= -\omega^2\vec{r}(t) \quad 0.5$$

$\vec{a}$ opposes $\vec{r}$, points to the centre of the circle 0.5

$$a_{cp} = |\vec{a}| = \sqrt{a_x^2 + a_y^2} = \omega^2|\vec{r}| = \omega^2 R \quad 1.0$$

why? $|\vec{r}(t)| = \sqrt{x(t)^2 + y(t)^2} = \sqrt{R^2\cos^2\omega t + R^2\sin^2\omega t}$

$= R\sqrt{\cos^2\omega t + \sin^2\omega t} = R$ (by trig relation $\sin^2 + \cos^2 = 1$)

From $\vec{v}(t)$ show: $v(t) = |\vec{v}(t)| = \sqrt{R^2\omega^2(\sin^2\omega t + \cos^2\omega t)}$

$= R\omega = v$ (const.) 1

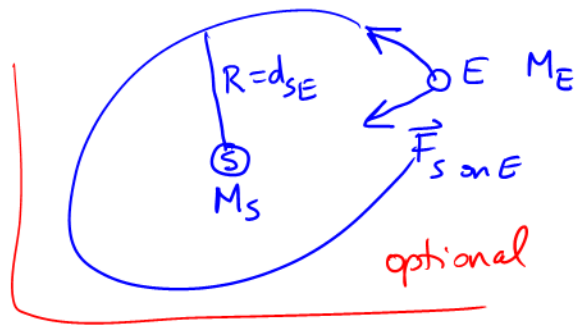
$$\therefore\ a_{cp} = \underset{0.5}{\omega^2 R} = \left(\frac{v}{R}\right)^2 R \underset{0.5}{=} \frac{v^2}{R}$$

[R is the constant radius of the circular motion and is denoted by $r = |\vec{r}(t)|$, i.e., $a_{cp} = \frac{v^2}{r}$]

optional

3) [5] Calculate the earth's linear speed in its motion around the sun starting from the law of gravity and Newton's 2nd law. Assume $d_{\text{S-E}} = 1.5 \times 10^{11}$ m, and $M_S = 2.0 \times 10^{30}$kg. Then calculate the length of a year from one orbit.

$$m\vec{a} = \vec{F}_{net} \longrightarrow \vec{F}_{S\,on\,E}$$ provides centripetal acc.



$$\cancel{M_E}\frac{v^2}{\cancel{d_{SE}}} = \frac{G\,\cancel{M_E}\,M_S}{d_{SE}^{\cancel{2}}}$$ 2

$$v^2 = \frac{G M_S}{d_{SE}} = \frac{6.67 \times 10^{-11} \cdot 2.0 \times 10^{30}}{1.5 \times 10^{11}} = 8.9 \times 10^{8}\ \frac{m^2}{s^2}$$

$$v = 3.0 \times 10^4\ \frac{m}{s} = 30\ \frac{km}{s}$$ 1

Orbit length: $s = 2\pi d_{SE}$; $s = vT$ 1

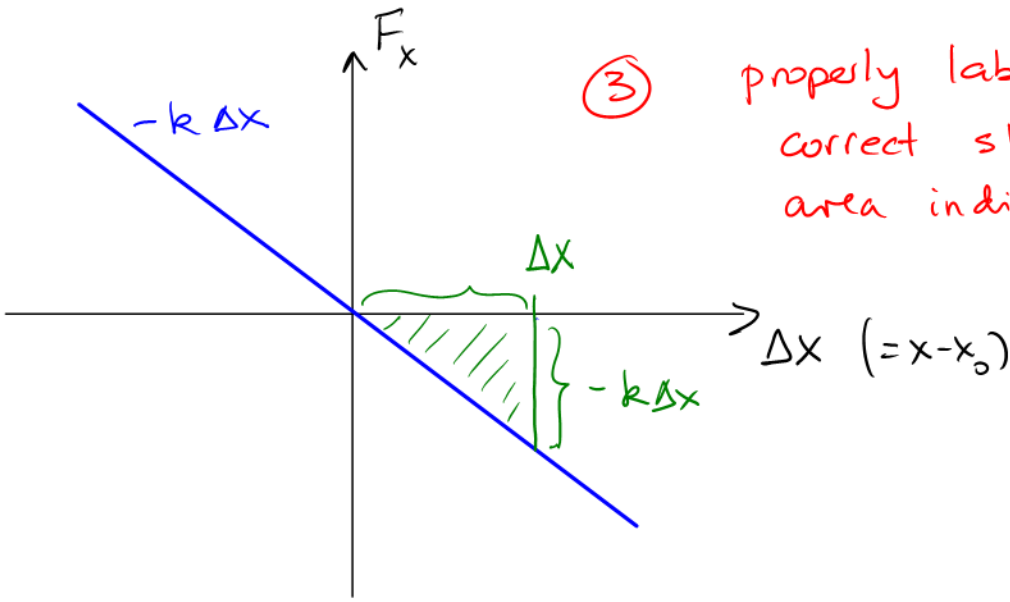
$$\therefore\ T = \frac{s}{v} = \frac{2\pi\, d_{SE}}{v} = 0.314 \times 10^{8}\ s$$
$$= 3.1 \times 10^{7}\ s$$ 1

makes sense?

$365 \times 24 \times 3600 = 31{,}536{,}000 \sim 3.2 \times 10^{7}$

optional

4) [5] When you compress a spring the force increases linearly with the displacement from equilibrium Δx. Calculate the work associated with this compression. Do the calculation based on geometry, do not use integral calculus, i.e., start with a graph of the spring force vs displacement Δx. By Hooke's law $F = -k\Delta x$, and note that Δx can be positive or negative.



3 properly labeled, correct slope (<0) area indicated

Area of the triangle: $\frac{1}{2}$(base × height)

$A = \frac{1}{2}\Delta x \cdot (-k\Delta x)$ 1

$= -\frac{1}{2}k\Delta x^2$

The work done by the spring force: $-\frac{1}{2}k\Delta x^2$ 0.5 0.5

["–" sign: we have to work against the spring] opt

FORMULA SHEET

$v(t_\mathrm{f}) = v(t_\mathrm{i}) + \int_{t_\mathrm{i}}^{t_\mathrm{f}} a(t)\,dt \qquad s(t_\mathrm{f}) = s(t_\mathrm{i}) + \int_{t_\mathrm{i}}^{t_\mathrm{f}} v(t)\,dt$

$v_\mathrm{f} = v_\mathrm{i} + a\Delta t \qquad s_\mathrm{f} = s_\mathrm{i} + v_\mathrm{i}\Delta t + \frac{1}{2}a\Delta t^2 \qquad v_\mathrm{f}^2 = v_\mathrm{i}^2 + 2a\Delta s \qquad g = 9.8\ \mathrm{m/s^2}$

$f(t) = t \quad \frac{df}{dt} = 1 \quad F(t) = \int f(t)\,dt = \frac{t^2}{2} + C$

$f(t) = a \quad \frac{df}{dt} = 0 \quad F(t) = \int f(t)\,dt = at + C \qquad F(t)$ = anti-derivative = indefinite integral

area under the curve $f(t)$ between limits t_1 and t_2: $F(t_2) - F(t_1)$

$x^2 + px + q = 0$ factored by: $x_{1,2} = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}$

uniform circular m. $\vec{r}(t) = R(\cos\omega t\ \hat{\mathbf{i}} + \sin\omega t\ \hat{\mathbf{j}})$; $\vec{v}(t) = \frac{d\vec{r}}{dt} = \ldots$; $\vec{a}(t) = \frac{d\vec{v}}{dt} = \ldots$.

$\exp' = \exp$; $\sin' = \cos$; $\cos' = -\sin$. $\quad \frac{d}{dx}[f(g(x))] = \frac{df}{dg}\frac{dg}{dx}$; $\quad (fg)' = f'g + fg'$

$m\vec{a} = \vec{F}_\mathrm{net}$; $\quad F_G = \frac{Gm_1m_2}{r^2}$; $g = \frac{GM_E}{R_E^2}$; $R_E = 6370$ km; $G = 6.67 \times 10^{-11}\frac{\mathrm{Nm^2}}{\mathrm{kg^2}}$; $M_E = 6.0 \times 10^{24}$kg

$f_\mathrm{s} \le \mu_\mathrm{s} n$; $\quad f_\mathrm{k} = \mu_\mathrm{k} n$; $\quad f_\mathrm{r} = \mu_\mathrm{r} n$; $\quad \mu_\mathrm{r} << \mu_\mathrm{k} < \mu_\mathrm{s}$. $\qquad F_H = -k\Delta x = -k(x - x_0)$.

$\vec{F}_\mathrm{d} \sim -\vec{v}$; linear: $F_\mathrm{d} = dv$; quadratic: $F_\mathrm{d} = 0.5\rho A v^2$; $\quad A$ = cross sectional area

$W = F\Delta x = F(\Delta r)\cos\theta \quad$ For $F(x)$ the work is given as area under the F_x vs x curve.