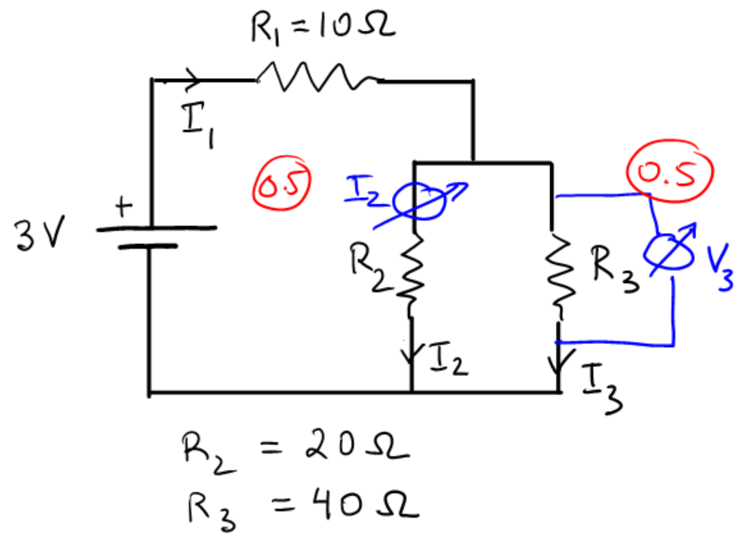
LAST NAME: STUDENT NR:

PHYS 1010 6.0: CLASS TEST 5

Time: 50 minutes; Calculators & formulae provided at the end = only aid; Total = 20 points.

1) [5] Given the following circuit diagram which contains an ideal battery, and three resistors, R_1, R_2, R_3. Complete the diagram by indicating how to measure: the current I_2 through resistor R_2; the voltage drop ΔV_3 across resistor R_3. Then calculate the currents I_1, I_2 and I_3, and the power dissipated by R_3.



Total current $I_1 = \dfrac{\Delta V_B}{R_{eq}}$

R_2 and R_3 are in parallel:

$$R_{23}^{eq} = \frac{R_2 R_3}{R_2 + R_3} = \frac{800}{60}\,\Omega = \frac{40}{3}\,\Omega$$

$$R_{eq} = R_1 + R_{23}^{eq} = \frac{70}{3}\,\Omega = 23.3\,\Omega$$

$$I_1 = \frac{3.0}{23.3} = 0.129\text{ A} \quad 1$$

$$\Delta V_1 = R_1 I_1 = 1.29\text{ V} \quad \therefore \quad \Delta V_2 = \Delta V_3 = (3 - 1.29)\text{ V} = 1.71\text{ V}$$

$$\therefore\ I_2 = \frac{\Delta V_2}{R_2} = 0.0855\text{ A} \quad 1 \quad , \quad I_3 = \frac{\Delta V_3}{R_3} = 0.04275\text{ A} \quad 1$$

$$P_3 = \Delta V_3 \cdot I_3 = 1.71\text{ V} \times 0.04275\text{ A} = 0.0731\text{ W} \quad 1$$

$$\therefore\ I_1 = 129\text{ mA}, \quad I_2 = 85.5\text{ mA}, \quad I_3 = 42.75\text{ mA}$$

$$P_3 = 73.1\text{ mW}$$

2) [5] The diagram shows a non-ideal battery (with internal resistance R_{int}), and a simple resistor-capacitor network. Assume that just before the switch is set from charging to discharging (at time $t = 0$) the capacitors are fully charged. Give a formula for the discharge current (without derivation, use the formula sheet which gives a generic expression). Calculate the time constant, and graph the current as a function of time (properly marked current and time axis is required!)

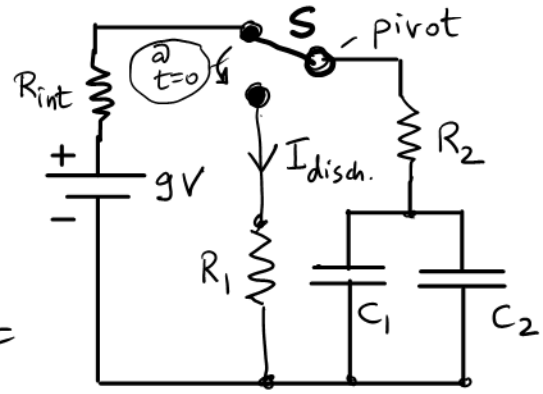
$R_{int} = 1.0\,\Omega$

$R_1 = 100\,\Omega$

$R_2 = 200\,\Omega$

$C_1 = 10\,\mu F$

$C_2 = 15\,\mu F$



C_1 and C_2 are in parallel $\therefore$ $C_{eq} = C_1 + C_2$

$C_{eq} = 25\,\mu F$ 0.5

The discharge is through R_1 and R_2 in series, i.e., $R_{eq} = R_1 + R_2$

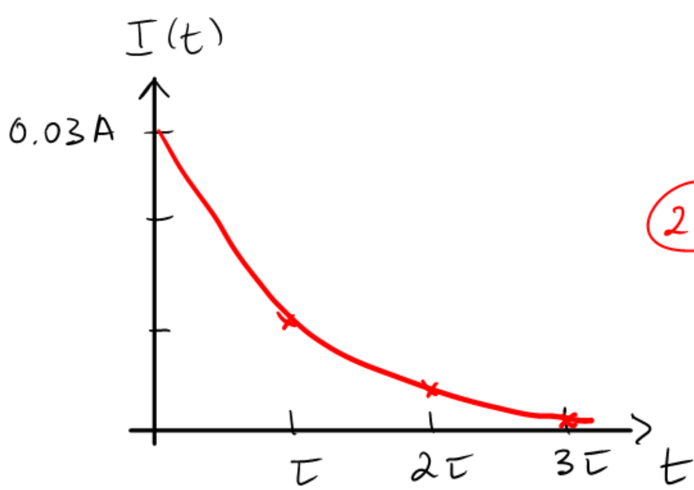
$R_{eq} = 300\,\Omega$ 0.5

$\tau = RC \Rightarrow R_{eq} \cdot C_{eq} = 300 \times 25 \times 10^{-6}\ \Omega F = 7500 \times 10^{-6}\,s$

$\tau = 7.5\,ms$ 1.0

$I(t) = I_0\, e^{-t/\tau}$ where $I_0 = \frac{\Delta V_C}{R_{eq}} = \frac{\Delta V_B}{R_{eq}} = \frac{9V}{300\,\Omega}$

$I_0 = 0.03\ A$ 1.0



$\tau = 7.5\,\mu s$

2.0

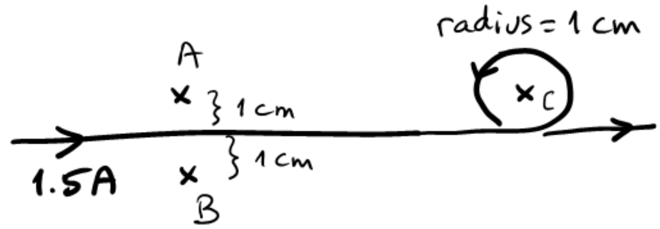
$\left(\frac{1}{e} \gtrsim \frac{1}{3}\right.$ is good enough!$\left.\right)$

0.5 for 0.03 A on y-axis @ t=0

0.5 for τ-value marked and 2τ, 3τ indicated

drawing the graph is also OK

3) [5] The figure shows a straight wire segment, and then a loop. The same current passes through. Three locations are marked, A, B are far away from the loop, so its contribution can be ignored. Location C is at the centre of the loop. Use the formulae provided to calculate the magnetic fields at A, B, C in the paper plane. A current of 1.5 A flows through the wire. Be careful with location C, there are two contributions. The fields are to be specified by magnitude and direction or by listing the appropriate component with sign!



By the RH rule the $\vec{B}$ field @ A is out of the plane ($B_z > 0$ if $\hat{j}$ $\hat{i}$ is implied) and @ B it is into 1 the plane. At C the contributions from the loop and from the straight wire add and give a stronger out-of-plane contribution.

A: $|B_z| = \frac{\mu_0}{2\pi}\frac{I}{d} = 2\times10^{-7}\ \frac{\mathrm{Tm}}{\mathrm{A}}\ \frac{1.5\ \mathrm{A}}{0.01\ \mathrm{m}} = 3.0\times10^{-5}\ \mathrm{T}$

$B_z = +3.0\times10^{-5}\ \mathrm{T}$ 1

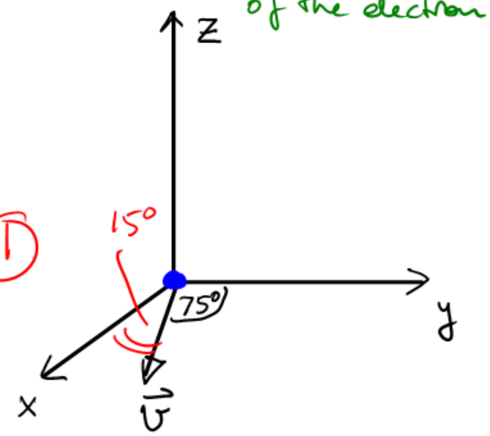
B: $B_z = -3.0\times10^{-5}\ \mathrm{T}$ 1

C: straight wire contributes same as at A, and also $B_{loop} = \frac{\mu_0 I}{2R}$ in the same direction

$B_{loop} = \frac{4\pi\times10^{-7}\ 1.5\ \mathrm{A}}{0.02\ \mathrm{m}}\ \frac{\mathrm{Tm}}{\mathrm{A}} = 9.42\times10^{-5}\ \mathrm{T}$ 1

$\therefore\quad B_z = +1.24\times10^{-4}\ \mathrm{T}$ 1 at (0,0,0)

4) [5] The particle in the figure has a negative charge, and its velocity vector lies in the $x-y$ plane and makes an angle of 75° with the y axis. A magnetic field is along the $+x$ direction. What is the direction of the magnetic force on the particle? Now you are told that the field has a strength of 1.5 T, and that the particle is an electron and its speed is $v = 500$ m/s. Calculate ~~the magnetic force.~~ and the acceleration of the electron



$$\vec{F}_M = q\, \vec{v} \times \vec{B}$$

By the RH rule $(\vec{B} = B_0\, \hat{i})$

$$\vec{v} \times \hat{i} \sim -\hat{k}$$

$\vec{v} \times \vec{B}$ is in the negative z dir'n

1

but $q < 0$, thus $\vec{F}_M \sim \hat{k}$, i.e., along positive z 1

$$F_{M,z} = +e\, |v|\, |B| \sin(15°)$$

$$\left(\text{or } -e\, |v|\, |B| \sin(360° - 15°)\right)$$ 1

$$F_{M,z} = 1.60 \times 10^{-19}\, C \times 500\, \frac{m}{s} \times 1.5\, T \times \underbrace{\sin(15°)}_{0.259}$$

$$= 3.11 \times 10^{-17}\, N$$

$$= 3.1 \times 10^{-17}\, N$$ 1

$$a_{e,z} = \frac{F_{M,z}}{m_e} = \frac{3.11 \times 10^{-17}}{9.11 \times 10^{-31}}\, \frac{N}{kg} = 0.34 \times 10^{14}\, \frac{m}{s^2}$$

$$= 3.4 \times 10^{13}\, \frac{m}{s^2}$$ 1

FORMULA SHEET

$v(t_\mathrm{f}) = v(t_\mathrm{i}) + \int_{t_\mathrm{i}}^{t_\mathrm{f}} a(t)\ dt \quad s(t_\mathrm{f}) = s(t_\mathrm{i}) + \int_{t_\mathrm{i}}^{t_\mathrm{f}} v(t)\ dt$

$v_\mathrm{f} = v_\mathrm{i} + a\Delta t \quad s_\mathrm{f} = s_\mathrm{i} + v_\mathrm{i}\Delta t + \frac{1}{2}a\Delta t^2 \quad v_\mathrm{f}^2 = v_\mathrm{i}^2 + 2a\Delta s \qquad g = 9.8\ \mathrm{m/s^2}$

$f(t) = t \quad \frac{df}{dt} = 1 \quad F(t) = \int f(t)\ dt = \frac{t^2}{2} + C$

$f(t) = a \quad \frac{df}{dt} = 0 \quad F(t) = \int f(t)\ dt = at + C \quad F(t) = \text{anti-derivative} = \text{indefinite integral}$

area under the curve $f(t)$ between limits t_1 and t_2: $F(t_2) - F(t_1)$

$x^2 + px + q = 0$ factored by: $x_{1,2} = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}$

uniform circular m. $\vec{r}(t) = R(\cos\omega t\ \hat{\mathbf{i}} + \sin\omega t\ \hat{\mathbf{j}})$; $\vec{v}(t) = \frac{d\vec{r}}{dt} = ...$; $\vec{a}(t) = \frac{d\vec{v}}{dt} = ...$

$\exp' = \exp$; $\sin' = \cos$; $\cos' = -\sin$. $\quad \frac{d}{dx}[f(g(x))] = \frac{df}{dg}\frac{dg}{dx}$; $\quad (fg)' = f'g + fg'$

$m\vec{a} = \vec{F}_\mathrm{net}$; $\quad F_G = \frac{Gm_1m_2}{r^2}$; $g = \frac{GM_E}{R_E^2}$; $R_E = 6370$ km; $G = 6.67 \times 10^{-11}\frac{\mathrm{Nm^2}}{\mathrm{kg^2}}$; $M_E = 6.0 \times 10^{24}$kg

$f_\mathrm{s} \le \mu_\mathrm{s} n$; $\quad f_\mathrm{k} = \mu_\mathrm{k} n$; $\quad f_\mathrm{r} = \mu_\mathrm{r} n$; $\quad \mu_\mathrm{r} << \mu_\mathrm{k} < \mu_\mathrm{s}$. $\qquad F_H = -k\Delta x = -k(x - x_0)$.

$\vec{F}_\mathrm{d} \sim -\vec{v}$; linear: $F_\mathrm{d} = dv$; quadratic: $F_\mathrm{d} = 0.5\rho A v^2$; $\quad A$ = cross sectional area

$W = F\Delta x = F(\Delta r)\cos\theta$. W = area under $F_x(x)$. $\quad PE_\mathrm{H} = \frac{k}{2}(\Delta x)^2$; $PE_g = mg\Delta y$.

$\Delta\vec{p} = \vec{J} = \int \vec{F}(t)dt$; $\Delta p_x = J_x$ = area under $F_x(t) = F_x^\mathrm{avg}\Delta t$; $\quad \vec{p} = m\vec{v}$; $K = \frac{m}{2}v^2$

$\Delta\vec{p}_1 + \Delta\vec{p}_2 = 0$; $K_1^\mathrm{in} + K_2^\mathrm{in} = K_1^\mathrm{fin} + K_2^\mathrm{fin}$ for elastic collisions. $\quad \vec{a}_\mathrm{CM} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1 + m_2}$

$\vec{\tau} = \vec{r} \times \vec{F}$; $\quad \tau_z = rF\sin(\alpha)$ for $\vec{r}$, $\vec{F}$ in xy plane. $\quad I = \sum_i m_i r_i^2$; $\quad I\alpha_z = \tau_z$; ($\hat{k}$ = rot. axis)

$K_\mathrm{rot} = \frac{I}{2}\omega^2$; $L_z = I\omega_z$; $\frac{d}{dt}L_z = \tau_z$; $\vec{L} = \vec{r} \times \vec{p}$; $\frac{d}{dt}\vec{L} = \vec{\tau}$

$x(t) = A\cos(\omega t + \phi)$; $\quad \omega = \frac{2\pi}{T} = 2\pi f$; $\quad v_x(t) = ...$; $\quad v_\mathrm{max} = ...$

$m_\mathrm{e} = 9.11 \times 10^{-31}$kg $\quad m_\mathrm{p} = 1.67 \times 10^{-27}$kg $\quad e = 1.60 \times 10^{-19}$C $\quad K = \frac{1}{4\pi\epsilon_0} = 9.0 \times 10^9\frac{\mathrm{Nm^2}}{\mathrm{C^2}}$

$\vec{F}_\mathrm{C} = \frac{Kq_1q_2}{r^2}\hat{\mathbf{r}} \quad \vec{F}_\mathrm{E} = q\vec{E} \quad E_\mathrm{line} = \frac{2K|\lambda|}{r} = \frac{2K|Q|}{Lr} \quad E_\mathrm{plane} = \frac{|\eta|}{2\epsilon_0} = \frac{|Q|}{2A\epsilon_0} \quad \vec{E}_\mathrm{cap} = \left(\frac{Q}{\epsilon_0 A}, \text{pos} \to \text{neg}\right)$

$\frac{mv^2}{2} + U_\mathrm{el}(s) = \frac{mv_0^2}{2} + U_\mathrm{el}(s_0)$, $(U \equiv PE_\mathrm{el}) \quad U_\mathrm{el} = qEx$ for $\vec{E} = -E\ \hat{i} \quad V_\mathrm{el} = U_\mathrm{el}/q \quad E_x = -\frac{dV_\mathrm{el}}{dx}$

$Q = C\Delta V_C \quad$ farad $= \mathrm{F} = \frac{\mathrm{C}}{\mathrm{V}} \quad C = \frac{\epsilon_0 A}{d} \quad \epsilon_0 = 8.85 \times 10^{-12}\frac{\mathrm{C^2}}{\mathrm{Nm^2}}$

parallel C_1, C_2: $C_\mathrm{eq} = C_1 + C_2 \quad$ series C_1, C_2: $C_\mathrm{eq}^{-1} = C_1^{-1} + C_2^{-1}$

$\Delta V_\mathrm{loop} = \sum_i \Delta V_i = 0 \quad \sum I_\mathrm{in} = \sum I_\mathrm{out}$

$P = \Delta V I \quad$ watt = W = VA $\quad P_R = \Delta V_R I = I^2 R$

$\tau = RC \quad Q(t) = Q_0 e^{-t/\tau} \quad I(t) = -\frac{dQ}{dt} = \frac{\Delta V_0}{R}e^{-t/\tau}$

$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\vec{r}}{r^3} = \frac{\mu_0}{4\pi}\frac{I\Delta\vec{s}\times\vec{r}}{r^3} \quad B_\mathrm{wire} = \frac{\mu_0}{2\pi}\frac{I}{d}$ (use RH rule) $\quad \frac{\mu_0}{4\pi} = 10^{-7}\frac{\mathrm{Tm}}{\mathrm{A}} \quad$ tesla $= \mathrm{T} = \frac{\mathrm{N}}{\mathrm{Am}}$

short coil, $R >> L$ (N turns): $B_\mathrm{coil,centre} = \frac{\mu_0 NI}{2R} \qquad$ solenoid, $L >> R$: $B_\mathrm{sol,inside} = \frac{\mu_0 NI}{L}$

mag dipole: $\vec{\mu} = (AI, \text{from south to north}) \quad \vec{B}_\mathrm{dip} = \frac{\mu_0}{4\pi}\frac{2\vec{\mu}}{z^3}$ on axis, far away

$\vec{F}_{\mathrm{on}q} = q\vec{v} \times \vec{B} \quad$ force on current $\perp$ to $\vec{B}$: $F_\mathrm{wire} = ILB$

force betw. parallel wires: $F_\mathrm{2wires} = \frac{\mu_0 L I_1 I_2}{2\pi d} \qquad$ torque on mag dipole: $\vec{\mu}$ in $\vec{B}$: $\vec{\tau} = \vec{\mu} \times \vec{B}$