LAST NAME: STUDENT NR:

PHYS 1010 6.0: CLASS TEST 6

Time: 50 minutes; Calculators & formulae provided at the end = only aid; Total = 20 points.

1) A current loop with a resistance $R = 250\ \Omega$ and an area $A = 0.4\ \text{m}^2$ is oriented perpendicular to a magnetic field that varies in time according to $B(t) = 0.5t(1-t)$ (t and B are in SI units). What is the current induced in the loop at $t = 0$ s, at $t = 0.5$ s, and at $t = 1$ s?

Faraday: $\mathcal{E} = -\frac{d\phi_M}{dt} = -\frac{d}{dt}\left(B(t)A\right)$ (orientation doesn't change) 1

$= -A\frac{dB}{dt} = -A\left(\frac{1}{2}(1-t) + \frac{1}{2}t(-1)\right)$

$\mathcal{E} = -\frac{A}{2}(1-2t)$ (Volts; in SI) 1

Ohm's law: $I = \frac{\mathcal{E}}{R} = \frac{2t-1}{500}\,0.4 = (2t-1)\,0.8\times10^{-3}$ 1 (A; in SI)

(orientation from Lenz, not required here)

a) $t=0$: -0.8 mA

b) $t=0.5$ s : 0 mA

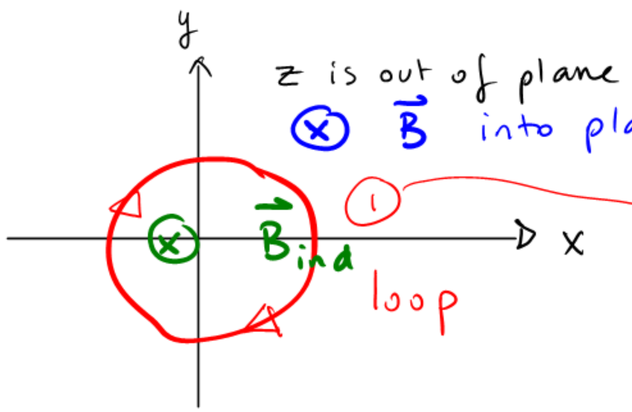
c) $t=1.0$ s : $+0.8$ mA

absolute sign doesn't matter, but a) relative to b) does, i.e., direction reversal!

1 for values

1 for relative sign

2) A circular loop of wire lies in the $x - y$ plane so that the axis of the loop lies along z. A homogeneous (spatially uniform) magnetic field $B(t)$ is anti-parallel to the z-axis. ($B_z(t) < 0$, $B_x = B_y = 0$). If $B(t)$ is decreasing with time, what is the direction of the induced current when viewed from above? Start with a drawing, and explain your steps and reasoning!



1

Lenz rule:

$\vec{B}_{ind}$ opposes change 1

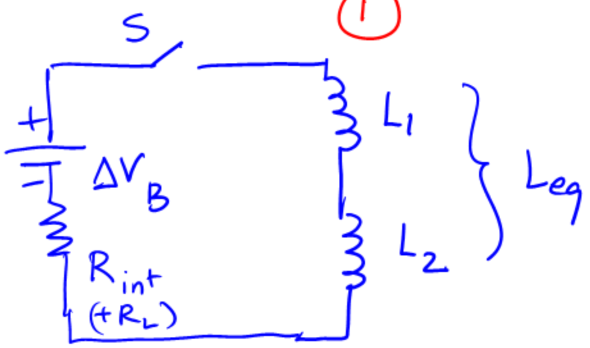
Since $\vec{B} = (0, 0, B_z(t))$ is pointing into the plane, but is decreasing, $\vec{B}_{ind}$ points into the plane 1

RH rule:

The direction of the current is CW 1 (as viewed from above)

real

3) Two inductors L_1 and L_2 are connected in series to form an equivalent inductor L_{eq}. Together they are connected to a real battery of voltage ΔV_B via a switch. Draw a circuit diagram depicting the situation. Formulate Kirchhoff's loop law for the circuit. Derive how L_{eq} depends on L_1 and L_2.



(0.5 for this)

$$\Delta V_B - RI - L_1 \frac{dI}{dt} - L_2 \frac{dI}{dt} = 0$$

1.5

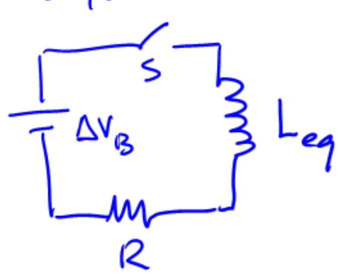
$\hookrightarrow R_{int} + R_L + R_{wire}$

(we should add $\underset{R}{\sim\!\!\sim}$ for the ohmic resistance in series in a realistic setting)

The least we need is R_{int} from the battery!

The same current passes through L_1 and L_2 $\Rightarrow$ the same $\frac{dI}{dt}$ acts in both $\leftarrow$ 0.5

Equivalent circuit:



1

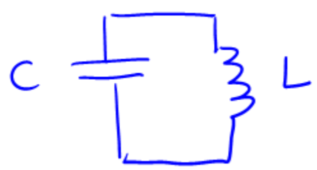
(can be shown as part of original circuit)

…

$$\left.\begin{aligned} \Delta V_B - RI - L_1\frac{dI}{dt} - L_2\frac{dI}{dt} &= 0 \\ \text{vs} \quad \Delta V_B - RI - L_{eq}\frac{dI}{dt} &= 0 \end{aligned}\right\}$$

$$L_{eq} = (L_1 + L_2) \quad 1$$

4a) Consider a simple LC circuit (L and C in parallel) with values $C = 1500$ pF and $L = 20$ mH. At $t = 0$ the current is $I = 45$ mA and the charge on C is zero. Calculate the energy stored in the inductor at this time. At which time t_1 will the current be zero (first occurrence after $t = 0$). What is the charge in C at this time? Provide an accurate sketch of the charge as a function of time for two complete cycles.



$I_0 = 45 \times 10^{-3}$ A

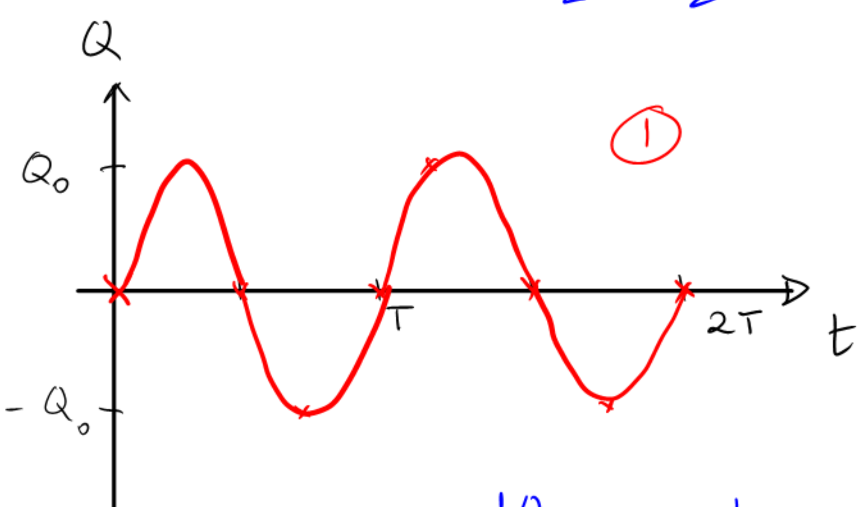
$C = 1.5 \times 10^{3} \times 10^{-12}$ F

$L = 2 \times 10^{-2}$ H

$PE_L = \frac{1}{2} L I^2 = 10^{-2} \times 45^2 \times 10^{-6}$ J (in SI)

$= 2.025 \times 10^{3} \times 10^{-8}$ J

$= 2.0 \times 10^{-5}$ J 0.5



Period of oscillation:

$\omega = \frac{1}{\sqrt{LC}} = \frac{2\pi}{T}$

$T = 2\pi\sqrt{LC} =$

$6.28 \times \sqrt{1.5\times10^{-9}\times 0.02}$ s

$= 0.344\ E{-4} = 3.4\times10^{-5}$ s

$= 34\ \mu s$ 0.5

Current $I = \frac{dQ}{dt}$ vanishes 0.5

when $t = T/4,\ \frac{3}{4}T, \ldots$

$t_1 = 8.6\ \mu s$ 0.5

Energy conservation: $\frac{C}{2}\Delta V^2 = \frac{L}{2} I^2$

$\Delta V_C = \frac{Q}{C} \quad \therefore \quad \frac{Q^2}{C} = L I^2 \quad \therefore \quad Q = \sqrt{LC}\, I = \sqrt{3.0\times10^{-11}} \times .045 = 25\ nC$ 0.5 0.5

4b) The LC circuit of question 4 has the inductor replaced by a $L = 20\ \mu$H coil. What is the wavelength of the radiowaves that this circuit will catch on resonance?

$$\omega = \frac{1}{\sqrt{LC}} = 2\pi f \qquad f = \frac{1}{2\pi\sqrt{LC}} = 0.92 \times 10^6\ \mathrm{Hz}$$

$$\lambda f = c \quad \therefore \quad \lambda = \frac{c}{f} = \frac{3.0 \times 10^8\ \mathrm{m/s}}{0.92 \times 10^6\ \mathrm{Hz}} = 326\ \mathrm{m}$$

$$\approx 330\ \mathrm{m}. \quad 1$$

FORMULA SHEET

$v(t_\mathrm{f}) = v(t_\mathrm{i}) + \int_{t_\mathrm{i}}^{t_\mathrm{f}} a(t)\ dt \quad s(t_\mathrm{f}) = s(t_\mathrm{i}) + \int_{t_\mathrm{i}}^{t_\mathrm{f}} v(t)\ dt$

$v_\mathrm{f} = v_\mathrm{i} + a\Delta t \quad s_\mathrm{f} = s_\mathrm{i} + v_\mathrm{i}\Delta t + \frac{1}{2}a\Delta t^2 \quad v_\mathrm{f}^2 = v_\mathrm{i}^2 + 2a\Delta s \qquad g = 9.8\ \mathrm{m/s^2}$

$f(t) = t \quad \frac{df}{dt} = 1 \quad F(t) = \int f(t)\ dt = \frac{t^2}{2} + C$

$f(t) = a \quad \frac{df}{dt} = 0 \quad F(t) = \int f(t)\ dt = at + C \quad F(t) =$ anti-derivative $=$ indefinite integral

area under the curve $f(t)$ between limits t_1 and t_2: $F(t_2) - F(t_1)$

$x^2 + px + q = 0$ factored by: $x_{1,2} = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}$

uniform circular m. $\vec{r}(t) = R(\cos\omega t\ \hat{\mathbf{i}} + \sin\omega t\ \hat{\mathbf{j}})$; $\vec{v}(t) = \frac{d\vec{r}}{dt} = ...$; $\vec{a}(t) = \frac{d\vec{v}}{dt} = ...$

$\exp' = \exp$; $\sin' = \cos$; $\cos' = -\sin$. $\quad \frac{d}{dx}[f(g(x))] = \frac{df}{dg}\frac{dg}{dx}$; $\quad (fg)' = f'g + fg'$

$m\vec{a} = \vec{F}_\mathrm{net}$; $\quad F_G = \frac{Gm_1m_2}{r^2}$; $g = \frac{GM_E}{R_E^2}$; $R_E = 6370$ km; $G = 6.67 \times 10^{-11} \frac{\mathrm{Nm^2}}{\mathrm{kg^2}}$; $M_E = 6.0 \times 10^{24}$kg

$f_\mathrm{s} \leq \mu_\mathrm{s} n$; $\quad f_\mathrm{k} = \mu_\mathrm{k} n$; $\quad f_\mathrm{r} = \mu_\mathrm{r} n$; $\quad \mu_\mathrm{r} << \mu_\mathrm{k} < \mu_\mathrm{s}$. $\qquad F_H = -k\Delta x = -k(x - x_0)$.

$\vec{F}_\mathrm{d} \sim -\vec{v}$; linear: $F_\mathrm{d} = dv$; quadratic: $F_\mathrm{d} = 0.5\rho A v^2$; $\quad A =$ cross sectional area

$W = F\Delta x = F(\Delta r)\cos\theta$. $\ W =$ area under $F_x(x)$. $\quad \mathrm{PE_H} = \frac{k}{2}(\Delta x)^2$; $\mathrm{PE}_g = mg\Delta y$.

$\Delta\vec{p} = \vec{J} = \int \vec{F}(t)dt$; $\Delta p_x = J_x =$ area under $F_x(t) = F_x^\mathrm{avg}\Delta t$; $\quad \vec{p} = m\vec{v}$; $K = \frac{m}{2}v^2$

$\Delta\vec{p}_1 + \Delta\vec{p}_2 = 0$; $K_1^\mathrm{in} + K_2^\mathrm{in} = K_1^\mathrm{fin} + K_2^\mathrm{fin}$ for elastic collisions. $\quad \vec{a}_\mathrm{CM} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1 + m_2}$

$\vec{\tau} = \vec{r} \times \vec{F}$; $\quad \tau_z = rF\sin(\alpha)$ for $\vec{r}$, $\vec{F}$ in xy plane. $\quad I = \sum_i m_i r_i^2$; $\quad I\alpha_z = \tau_z$; ($\hat{k} =$ rot. axis)

$K_\mathrm{rot} = \frac{I}{2}\omega^2$; $L_z = I\omega_z$; $\frac{d}{dt}L_z = \tau_z$; $\vec{L} = \vec{r} \times \vec{p}$; $\frac{d}{dt}\vec{L} = \vec{\tau}$

$x(t) = A\cos(\omega t + \phi)$; $\quad \omega = \frac{2\pi}{T} = 2\pi f$; $\quad v_x(t) = ...$; $\quad v_\mathrm{max} = ...$

$m_\mathrm{e} = 9.11 \times 10^{-31}$kg $\quad m_\mathrm{p} = 1.67 \times 10^{-27}$kg $\quad e = 1.60 \times 10^{-19}$C $\quad K = \frac{1}{4\pi\epsilon_0} = 9.0 \times 10^9 \frac{\mathrm{Nm^2}}{\mathrm{C^2}}$

$\vec{F}_\mathrm{C} = \frac{Kq_1q_2}{r^2}\hat{\mathbf{r}} \quad \vec{F}_\mathrm{E} = q\vec{E} \quad E_\mathrm{line} = \frac{2K|\lambda|}{r} = \frac{2K|Q|}{Lr} \quad E_\mathrm{plane} = \frac{|\eta|}{2\epsilon_0} = \frac{|Q|}{2A\epsilon_0} \quad \vec{E}_\mathrm{cap} = \left(\frac{Q}{\epsilon_0 A}, \mathrm{pos} \rightarrow \mathrm{neg}\right)$

$\frac{mv^2}{2} + U_\mathrm{el}(s) = \frac{mv_0^2}{2} + U_\mathrm{el}(s_0)$, $(U \equiv PE_\mathrm{el}) \quad U_\mathrm{el} = qEx$ for $\vec{E} = -E\,\hat{i} \quad V_\mathrm{el} = U_\mathrm{el}/q \quad E_x = -\frac{dV_\mathrm{el}}{dx}$

$Q = C\Delta V_C$ farad $= \mathrm{F} = \frac{\mathrm{C}}{\mathrm{V}}$ $C = \frac{\epsilon_0 A}{d}$ $\epsilon_0 = 8.85 \times 10^{-12} \frac{\mathrm{C}^2}{\mathrm{Nm}^2}$ $\mathrm{PE}_C = \frac{C}{2} \Delta V_C{}^2$

parallel C_1, C_2: $C_{\mathrm{eq}} = C_1 + C_2$ series C_1, C_2: $C_{\mathrm{eq}}^{-1} = C_1^{-1} + C_2^{-1}$

$\Delta V_{\mathrm{loop}} = \sum_i \Delta V_i = 0$ $\sum I_{\mathrm{in}} = \sum I_{\mathrm{out}}$

$P = \Delta V I$ watt $= \mathrm{W} = \mathrm{VA}$ $P_R = \Delta V_R I = I^2 R$

$\tau = RC$ $Q(t) = Q_0 e^{-t/\tau}$ $I(t) = -\frac{dQ}{dt} = \frac{\Delta V_0}{R} e^{-t/\tau}$

$\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \vec{r}}{r^3} = \frac{\mu_0}{4\pi} \frac{I \Delta \vec{s} \times \vec{r}}{r^3}$ $B_{\mathrm{wire}} = \frac{\mu_0}{2\pi} \frac{I}{d}$ (use RH rule) $\frac{\mu_0}{4\pi} = 10^{-7} \frac{\mathrm{Tm}}{\mathrm{A}}$ tesla $= \mathrm{T} = \frac{\mathrm{N}}{\mathrm{Am}}$

short coil, $R >> L$ (N turns): $B_{\mathrm{coil,centre}} = \frac{\mu_0 N I}{2R}$ solenoid, $L >> R$: $B_{\mathrm{sol,inside}} = \frac{\mu_0 N I}{L}$

mag dipole: $\vec{\mu} = (AI, \text{from south to north})$ $\vec{B}_{\mathrm{dip}} = \frac{\mu_0}{4\pi} \frac{2\vec{\mu}}{z^3}$ on axis, far away

$\vec{F}_{\mathrm{on}q} = q\vec{v} \times \vec{B}$ force on current $\perp$ to $\vec{B}$: $F_{\mathrm{wire}} = ILB$

force betw. parallel wires: $F_{\mathrm{2wires}} = \frac{\mu_0 L I_1 I_2}{2\pi d}$ torque on mag dipole: $\vec{\mu}$ in $\vec{B}$: $\vec{\tau} = \vec{\mu} \times \vec{B}$

bar (length L) moves w. $\vec{v} \perp \vec{B}$ gen. EMF: $\varepsilon = vLB$;

$\Phi_m = \vec{A} \cdot \vec{B}$ $\Phi_m = AB\cos\theta$ $\varepsilon = \left|\frac{d\Phi_m}{dt}\right| = \left|\vec{B} \cdot \frac{d\vec{A}}{dt} + \vec{A} \cdot \frac{d\vec{B}}{dt}\right|$

$L = \frac{\Phi_m}{I}$ henry $= \mathrm{H} = \frac{\mathrm{Tm}^2}{\mathrm{A}}$ $\varepsilon_{\mathrm{coil}} = L\left|\frac{dI}{dt}\right|$ $\Delta V_L = -L\frac{dI}{dt}$ $\mathrm{PE}_L = \frac{L}{2} I^2$

series L and R: $\tau = \frac{L}{R}$ $I(t) = I_0 e^{-t/\tau}$; parallel L and C: $\omega = \sqrt{\frac{1}{LC}}$ $I(t) = \omega Q_0 \sin\omega t$

$\lambda f = v_{\mathrm{w}}$ sinusoid travelling in pos dir'n: $D(x,t) = A\sin(2\pi(\frac{x}{\lambda} - \frac{t}{T}) + \phi_0) = A\sin(kx - \omega t + \phi_0)$

transverse wave on a string: $v_{\mathrm{w}} = \sqrt{\frac{T}{\mu}}$ where T is tension, $\mu = M/L$

$\omega = v_{\mathrm{w}} k$ $v_{\mathrm{sound}} = 343\mathrm{m/s}$ in air at $\mathrm{T} = 20°\mathrm{C}$ in water: $v_{\mathrm{sound}} = 1480\mathrm{m/s}$

light in vac.: $v_{\mathrm{w}} = c = 3.00 \times 10^8\mathrm{m/s}$ visible: $\lambda = 400\mathrm{nm}$ (blue/UV); $\lambda = 700\mathrm{nm}$ (red/IR)

refraction: $n_{\mathrm{glass}} = 1.5$; $n_{\mathrm{water}} = 1.333$; light in medium: c/n; wavelength: λ_{vac}/n

approaching source (speed v_{s}): $f_+ = \frac{f_0}{1 - v_{\mathrm{s}}/v_{\mathrm{w}}}$ receding: $f_- = \frac{f_0}{1 + v_{\mathrm{s}}/v_{\mathrm{w}}}$

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$ $\sin\alpha + \sin\beta = 2\cos\frac{\alpha - \beta}{2} \sin\frac{\alpha + \beta}{2}$

transverse standing wave, string length L: $\lambda_n = \frac{2L}{n}$ $n = 1, 2, ...$ f_n from $\lambda_n f_n = c_{\mathrm{w}}$